Return Calculation of U.S. Treasury Constant Maturity Indices

Morningstar Methodology Paper

September 30, 2008

Content

Introduction

The Federal Reserve Board publishes a group of Treasury bond interest rates of various maturities. These are named Treasury Constant Maturities, the best known being the Ten-Year Treasury Constant Maturity. The interest rate, also known as the yield, of the Ten-Year Treasury Constant Maturity is often used as a reference point in valuation of other types of bonds such as corporate, municipal, etc.

The yields are interpolated by the U.S. Treasury from the daily yield curve. This curve, which relates the yield on a security to its time to maturity, is based on the closing market bid yields on actively traded Treasury securities in the over-the-counter market. These market yields are calculated from composites of quotations obtained by the Federal Reserve Bank of New York. The constant maturity yield values are read from the yield curve at fixed maturities, currently one, three and six months and one, two, three, five, seven, 10, 20, and 30 years. All constant maturity yields are quoted on a yield-to-maturity basis regardless of maturity, and the day count is based on actual over 365 or 366 days a year.

For benchmark comparison and various other purposes, knowing the yield is often insufficient, and the return information is required. However, based on the nature of the yield being interpolated from the yield curve, insufficient information exists to make a precise calculation of returns possible, but approximations can be estimated by making assumptions. This methodology addresses the assumptions and formulas used in calculating the Total Return, Capital Appreciation, and Income Return. This document is divided into two sections to separately address indices with one or more years to maturity from those with less than one year to maturity.

Indices with One or More Years to Maturity

Assumptions
Following are the assumptions that Morningstar makes in the return calculation of the U.S. Treasury Constant Maturity indices with one or more years to maturity:

1. Each index consists of a single coupon bond.
2. At the beginning of each month a bond is purchased at the prior month-end price, and daily returns in the month reflect the change in daily valuation of this bond.
3. Coupon is paid on the month-end day of every six months from the purchase day.
4. Each bond is trading at par upon purchase.
5. The yield curve is flat at the desired time to maturity.

Four factors drive a coupon bond's price, and they are the yield, the time to maturity, the coupon payments, and the redemption or face value. The coupon rates of these indices are not provided by the Federal Reserve Board, and we assume that the coupon rate is the same as the yield by assuming that the bond is trading at par. By definition, a bond that is trading at par is priced at 100, and its yield is the same as the coupon rate.

These indices' yields at the end of the holding period are not available. For example, at the end of a one-month holding period, a bond that had one year to maturity at the beginning of the holding period now has 11 months to maturity, and the Federal Reserve Board does not publish the yield of a 11-month bond. By assuming that the yield curve is flat at this segment, the yield of a newly published one-year bond is used as the yield of the old bond that has 11 months left to maturity.

These assumptions are reflected in formula [4]. The price of a bond is has two major components in the formula. The first component reflects the discounted face value of a bond, and the second component represents the present value of coupon payments. The yield of the bond on the purchase date is set equal to the coupon rate in the second component of formula [4] so that the bond is at par upon purchase.

Indices with One or More Years to Maturity (continued)

Return Formula

A bond is purchased at the beginning of each month at the prior month-end price, as stated in the Assumptions. This bond's price is tracked daily during the month to arrive at the daily total return of the bond. At the end of the month this bond is sold, and a new bond is purchased for next month. The purchase date of the bond, denoted as p, is the prior month-end of the desired month. For example, to calculate the price of the bond on any day in the month of January 2008, the purchase date of the bond is December 31, 2007. The maturity date, denoted as m, reflects the maturity of the index at purchase. For example, for the ten-year constant maturity index, the bond purchased on December 31, 2007 has a maturity date of December, 31, 2017. In the formulas below, a bond is identified by its maturity date.

[1] $$TR_{t_1,t_2} = \frac{P(t_2, y_{t_2,m}, m)}{P(t_1, y_{t_1,m}, m)} - 1$$

[2] $$IR_{t_1,t_2} = \frac{P(t_2, y_{p,m}, m)}{P(t_1, y_{p,m}, m)} - 1$$

[3] $$CA_{t_1,t_2} = TR_{t_1,t_2} - IR_{t_1,t_2}$$

Where:

TR_{t_1,t_2}	=	total return for the holding period from t_1 to t_2
IR_{t_1,t_2}	=	income return for the holding period from t_1 to t_2
CA_{t_1,t_2}	=	capital appreciation, also known as price return, for the holding period from t_1 to t_2
$P(t, y, m)$	=	price of the bond with maturity date "m", yield "y", at time "t". See formula [4].

Indices with One or More Years to Maturity (continued)

Return Formula (continued)

Note:

- ► $p \le t_1 < t_2 \le (p +$ one month). In other words, p denotes the purchase date of the bond, and both t_1 and t_2 must be within the one-month holding period of this bond. For example, for the bond that is purchased on December 31, 2007, t_2 can be any day in the month of January 2008. And t_1 can be on or after December 31, 2007 and before t_2 .
- ► If t is not a business day, its yield is the yield of the prior business day.

Indices with One or More Years to Maturity (continued)

Price Formula

[4]

$$P(t,y,m)=\left[\frac{100}{\left(1+\frac{y}{2}\right)^{2N_m-1+\frac{D_{t,m}}{S_m}}}\right]+\left\{100\frac{y_{p,m}}{y}\left(1+\frac{y}{2}\right)^{1-\frac{D_{t,m}}{S_m}}\left[1-\left(1+\frac{y}{2}\right)^{-2N_m}\right]\right\}$$

[5] $$D_{t,m}=c_m-t$$

[6] $$S_m=c_m-p$$

Where:

$P(t,y,m)$	=	price of the bond with maturity date "m", yield "y", at time "t"
y	=	Yield, also known as interest rate, expressed in decimal format
N_m	=	maturity of the bond, expressed in number of years
$D_{t,m}$	=	number of days between time "t" and the next coupon date of the bond
S_m	=	number of days in the coupon period in which time "t" falls, which is the first coupon period
$y_{p,m}$	=	coupon rate expressed in decimal format, and it is the yield of the bond on its purchase day "p"
c_m	=	day of first coupon payment, which is the month-end day six months from purchase day "p"

Indices with One or More Years to Maturity (continued)

Month-End Pricing
At month-end there are two different bonds because this is the day when the old bond that has been held during the month is sold, and a new bond is purchased for next month's use. Therefore, at month-end there are two prices, one representing each bond.

For example, on 1/31/2008, the bond held during January 2008 is sold. Let us identify this bond with its maturity of m_1, and its price is calculated using a coupon rate that is the yield on its purchase date of 12/31/2007. This price is used as the numerator, and the price of bond m_1 on 1/30/2008 serves as the denominator in calculating the daily return for 1/31/2008.

In addition, on 1/31/2008, a new bond is purchased to be held during February 2008, and let us identify this bond with its maturity of m_2. This bond's price is 100 on 1/31/2008 because it is a par bond. This price serves as the denominator to the price of bond m_2 on 2/1/2008 in order to produce the daily return for 2/1/2008.

Indices with One or More Years to Maturity (continued)

Calculation Steps and Example

The following example calculates the returns of the One-Year U.S. Treasury Constant Maturity index assuming that the holding period is one day. Return is calculated for January 15, 2008 to illustrate an example in a leap year. Time t_2 is 1/15/2008, time t_1 is 1/14/2008, purchase day p is 12/31/2007, and the next coupon payment day c_m is on the month-end day that is six months from purchase day and is 6/30/2008. The steps are as follow:

1. Obtain the yield of the One-Year U.S. Treasury Constant Maturity index on 12/31/2007, 1/14/2008, and 1/15/2008.
2. Calculate $P(t_2, y_{t_2,m}, m)$ using formula [4] by plugging in the 1/15/2008 yield as y, entering $N_m = 1$, $D_{t,m} = 167$, $S_m = 182$, and using the 12/31/2007 yield as $y_{p,m}$.
3. Calculate $P(t_1, y_{t_1,m}, m)$ using formula [4] by plugging in the 1/14/2008 yield as y, entering $N_m = 1$, $D_{t,m} = 168$, $S_m = 182$, and using the 12/31/2007 yield as $y_{p,m}$.
4. Calculate the 1/15/2008 total return using equation [1] by plugging in $P(t_2, y_{t_2,m}, m)$ and $P(t_1, y_{t_1,m}, m)$ from steps #2 and #3 above.
5. Determine $P(t_2, y_{p,m}, m)$ using formula [4] by plugging in the 12/31/2007 yield as y, entering $N_m = 1$, $D_{t,m} = 167$, $S_m = 182$, and using the 12/31/2007 yield as $y_{p,m}$.
6. Determine $P(t_1, y_{p,m}, m)$ using formula [4] by plugging in the 12/31/2007 yield as y, entering $N_m = 1$, $D_{t,m} = 168$, $S_m = 182$, and using the 12/31/2007 yield as $y_{p,m}$.
7. Determine the 1/15/2008 income return using formula [2] by plugging in $P(t_2, y_{p,m}, m)$ and $P(t_1, y_{p,m}, m)$ from steps #5 and #6 above.
6. Calculate the 1/15/2008 capital appreciation using formula [3] by plugging in the total return and income return derived from steps #4 and #7 above.

Indices with Less Than One Year to Maturity

Assumptions
Following are the assumptions that Morningstar makes in the return calculation of the U.S. Treasury Constant Maturity indices with less than one year to maturity:

1. Each index consists of a single bill that does not pay coupons.
2. At the beginning of each month a bill is purchased at the prior month-end price, and daily returns in the month reflect the change in daily valuation of this bill.
3. The yield curve is flat at the desired time to maturity.

These indices' yields at the end of the holding period are not available. For example, at the end of a one-month holding period, a bill that had six months to maturity at the beginning of the holding period now has five months to maturity, and the Federal Reserve Board does not publish the yield of a five-month bill. By assuming that the yield curve is flat at this segment, the yield of a newly published five-month bill is used as the yield of the old bill that has five months left to maturity.

Indices with Less Than One Year to Maturity (continued)

Formula

[7] $$TR_{t_1,t_2} = \frac{P(t_2, y_{t_2,m}, m)}{P(t_1, y_{t_1,m}, m)} - 1$$

[8] $$P(t, y, m) = \frac{100}{\left(1 + \frac{y}{2}\right)^{(m-t)/d}}$$

Where:

TR_{t_1,t_2}	=	total return for the holding period from t_1 to t_2
$P(t, y, m)$	=	price of the bond with maturity date "m", yield "y", at time "t"
y	=	Yield, also known as interest rate, expressed in decimal format
m	=	maturity date of the bill
t	=	time "t"
d	=	number of days between the purchase date and the month-end day six months from the purchase day (i.e. this is the number of days in the ½ year from the bond's purchase)

Note:

- $p \le t_1 < t_2 \le$ (p + one month). In other words, p denotes the purchase date of the bill, and both t_1 and t_2 must be within the one-month holding period of this bill. For example, for the bill that is purchased on December 31, 2007, t_2 can be any day in the month of January 2008. And t_1 can be on or after December 31, 2007 and before t_2 .
- If t is not a business day, its yield is the yield of the prior business day.
- This pricing formula is different from the discount formula often associated with bills. This is because the Federal Reserve Board converts the yield of bills quoted in discount format to the coupon-equivalent form prior to constructing the U.S. Treasury Yield Curve so that yields on the entire curve are quoted on a yield-to-maturity basis.