



Stock Grade Methodology for Financial Health

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Introduction

Morningstar continuously strives to improve its algorithms and to enhance its tools and their application in the investment decision-making process. Key algorithms and tools are frequently reviewed for quality assurance as well as updates and enhancements. The enhancements are initiated based on both internal peer reviews and Morningstar customers' inputs.

The upgraded and enhanced algorithm for stock grading in this document consists of five (5) grades (A, B, C, D, and F) for financial health. Grades will be assigned based on relative rankings of companies' distance to default scores within the qualified universe of Morningstar stocks.

All underlying financial data used to calculate the company's distance to default scores are from Morningstar's internal equities database. Market data required for estimation of the distance to the default scores, including stock prices and the safe rate (one-year Treasury yield), are downloaded from current feeds available to Morningstar.

Theory

Instead of using accounting-based ratios to formulate a measure to reflect the financial health of a firm, as per the previous Morningstar algorithm, the new approach makes use of structural or contingent claim models. The reason for the switch to a structural model is partly based on the fact that any accounting-based algorithm is handicapped from the implicit assumption of the going-concern principle, which assumes a firm is not going to go bankrupt precluding a robust assessment of the firm's financial health strictly based on the firm's reported financial statements. More so, in some cases accounting measures based on historical financial statements may have little or no bearing on the future viability of a firm. An area of weakness in accounting measures based on historical financial statements of the firm, for example, is the inherent understatement of the firm's book value in most cases, which results in overstatement of the financial leverage measures if such are to be used as a proxy of the financial health of the firm. Lastly, accounting measures based on historical financial statements of the firm do not account for the volatility of the firm's assets in spite the fact that the probability of bankruptcy and the volatility of the firm's assets are related. For example, two firms having identical financial leverage can have significantly differing probabilities of bankruptcy depending on their assets' volatility.

Structural Models

Structural models take advantage of both market information and accounting financial information. For this purpose, option pricing models based on seminal works of Black and Scholes (1973) and Merton (1974) are a natural fit. The firm's equity can be viewed as a call option on the value of the firm's assets. If the value of the assets is not sufficient to cover the firm's liabilities (the strike price), default is expected to occur and the call option expires worthless and the firm is turned over to its creditors.

$$\text{Asset Value} = \text{Market Value} + \text{Total Liabilities} \quad (1)$$

The underlying premise of contingent claim models is that default occurs when the value of the firm's assets falls below a certain threshold level in relation to the firm's liabilities. According to Merton (1974) if the firm's liabilities consist of one zero-coupon bond with notional value L, maturing in T (without any debt payment until T), and equity holders wait until T (to benefit from the expected increase of the asset value), the default probability, at time T, is that the value of the assets is less than the value of the liabilities. To estimate this probability, the value of the firm's liability is obtained from the firm's latest balance sheet.

$$L_t = L_s + L_l \quad (2)$$

Where:

$L_t = \text{Total Liabilities}$

$L_s = \text{Short Term Liabilities}$

$L_l = \text{Long Term Liabilities}$

Next, the probability distribution of the value of the firm's assets at time T needs to be estimated. It is assumed that the value of the firm's assets follows a log-normal distribution, i.e., the logarithm of the firm's asset value is normally distributed and the expected change in log asset values is $\mu - \delta - \sigma_A^2 / 2$. The log asset value in T year therefore follows a normal distribution with the following parameters:

$$\text{Ln}(A_T) \approx N[\text{Ln}(A_t) + (\mu - \delta - \sigma_A^2 / 2)(T - t), \sigma_A^2(T - t)] \quad (3)$$

Where

μ is the continuously compounded expected return on assets or the asset drift and

δ is the asset yield, expressed in terms of current asset value and is equal to
(TTM common + preferred dividends) / Current asset value

Next, the probability that a normally distributed variable x falls below z is given by

$N\{(z - E[x]) / \sigma_A(x)\}$ Where N denotes the cumulative standard normal distribution.

To empirically estimate the Black-Scholes probability from equation (3), we need estimates of A_t , μ , and σ_A , which are not directly observable. If they were known there would be no need to use Black-Scholes and the probability of bankruptcy (PB) and distance to default (DD) could be expressed (McDonald 2002) as:

$$PB = N\{-(\text{Ln } A_t / \text{Ln } L_t + (\mu - \delta - \sigma_A^2) (T - t)) / (\sigma_A \sqrt{(T - t)})\} \quad (4)$$

$$DD = (\text{Ln } A_t + (\mu - \delta - \sigma_A^2 / 2)(T - t) - \text{Ln } L_t) / \sigma_A \sqrt{(T - t)} \quad (5)$$

$$\Rightarrow PB = N[-DD] \quad (6)$$

Equation (4) shows that probability of bankruptcy is a function of the distance between the value of firm's assets today and the book value of firm's total liabilities, (A_t / L_t) adjusted for the expected growth in asset value, asset drift, and asset yield, $(\mu - \delta - \sigma_A^2 / 2)$ relative to asset volatility, σ_A .

But the market value of the firm's assets is not observable and can be very different than the book value of the firm's assets. This is the A_t in equation (4). Furthermore, we do not know the volatility of the market value of the firm's assets, nor can we use the observed asset values (book values) as a proxy for the volatility of the firm's market value of assets, σ_A . That's where the option pricing model comes in: It implies a relationship between the unobservable (A_t, σ_A) and the observable variables. As long as the value of the firm's assets is below the book value of the firm's liabilities, the payoff to equity holders is zero. If the value of the firm's assets is higher than book value of the firm's liabilities, equity holders receive the residual value and their payoff increases linearly as the value of the firm's assets increases over time. This can be expressed as payoff of a modified (for dividends) European call option:

$$E_T = \text{Max}(0, A_T - L_t) \quad (7)$$

Assuming risk-neutrality, the equity value, E_t , can be estimated by a modified (for dividends) standard Black-Scholes call option formula:

$$E_t = A_t e^{-\delta T} * N(d_1) - L_t * e^{-rT} * N(d_2) + (1 - e^{-\delta T}) A_t \quad (8)$$

Where

r is the safe rate (one-year Treasury yield), and

$N(d_1)$ and $N(d_2)$ are the cumulative standard normal distribution of d_1 and d_2 .

Dividend yield, δ , is added to the standard Black-Scholes model in Equation (8); it appears twice in the right-hand side of the equation. First, term $A_t e^{-\delta T}$ accounts for the reduction in the value of firm's assets due to dividends that are distributed at time T. Second, term $(1 - e^{-\delta T}) A_t$ accounts for the fact that it is the equity holders who receive the dividend--these terms do not appear in the standard Black-Scholes equation for valuing a call option on a dividend-paying stock because dividends are not paid to option holders:

$$d_1 = [\text{Ln}(A_t / L_t) + (r - \delta - \sigma_A^2 / 2)) T] / \sigma_A \sqrt{T} \quad (9)$$

and

$$d_2 = d_1 - \sigma_A \sqrt{T} = [\text{Ln}(A_t / L_t) + (r - \delta - \sigma_A^2)) T] / \sigma_A \sqrt{T} \quad (10)$$

Given the assumption of risk-neutrality, the value of the call option derived from the standard Black-Scholes formula is not a function of firm's asset return or drift, μ . The risk-neutrality assumption in the Black-Scholes formula implies that assets are expected to grow at the safe rate of return and therefore only the risk-free rate, r , enters the Black-Scholes equation. The

actual probability of bankruptcy, however, depends on the actual distribution of future values of assets and is a function of firm's asset drift, μ , as per modified Black-Scholes Equation (4).

The objective is the estimation of the firm's value of assets, A_t , drift, μ , and volatility, σ_A , though we only have one Equation (8) establishing a link between the two unknown values A_t and σ_A .

There are different methods to obtain more information to estimate these two values. One approach is to come up with another equation that establishes additional link between these two values. Then both equations can be simultaneously solved to determine these values. The optimal hedge Equation (11), which shows the equity volatility, σ_E , is related to asset value, A_t , and asset volatility, σ_A , establishes the additional relationship between the two values. Again d_1 in Equation (11) is the standard Black-Scholes d_1 per equation (9). Terms $A_t e^{-\delta T}$ in Equation (11) is needed to reflect the reduction in the value of the firm's assets due to dividends that are distributed at time T:

$$\sigma_E = (A_t e^{-\delta T} N(d_1) \sigma_A) / E_t \quad (11)$$

If we know the equity value, E_t (market price times shares outstanding), and we have an estimate of equity volatility, σ_E (annualized standard deviation of stocks' daily log returns), Equations (8) and (11) contain two unknowns (A_t , σ_A) that can be solved simultaneously for a numeric solution of the firm's asset value.

Alternatively, the firm's asset value, drift, and volatility can be estimated iteratively based on daily calculations of asset values and the use of the capital asset pricing model (CAPM). By rearranging Equation (8) we obtain asset value A_t :

$$A_t = \frac{E_t + L_t e^{-rT} N(d_2)}{1 - e^{-\delta T} + e^{-\delta T} N(d_1)} \quad (12)$$

The most current asset value estimated in equation (12), asset drift and asset volatility can then be directly entered into the distance to default and probability of default in Equations (4) and (5).

Calculations

Overview

The following example illustrates step-by-step the formulation and solution for the iterative method.

- 1) Set the time horizon $T - t = 1$ year (actual trailing 12 months, or TTM business days)

- 2) Set the daily equity value, E_t , for the TTM by multiplying the daily common stock price by shares outstanding.
- 3) Set total daily liabilities, L_t , equal to the latest available quarterly sum of short-term liabilities, L_s , and long-term liabilities, LI , for the TTM -- note these figures remain the same for each day and change only when a newer quarterly balance sheet becomes available during the TTM period.
- 4) Set the daily gross common and preferred dividends paid in the TTM--use the record date instead of the payout date and calculate TTM dividends and the annual rate of daily asset yields.
- 5) Set the daily yield for one-year Treasuries for the TTM.
- 6) Calculate the daily asset values and their volatility for the TTM.
- 7) Calculate the daily asset yield using TTM dividends divided by the daily asset value calculated from step 6.

A firm's asset volatility is calculated as the annualized standard deviation of the preceding TTM (approximately 252 business days) business daily log returns of asset values. To calculate the daily log returns of asset values for the TTM period we simply take the natural log of day two asset value divided by the natural log of day one asset value and repeat the process for all 252 business days. Next we estimate asset drift, μ , using CAPM. To do this, first, asset beta is calculated as the log of slope of regression line for excess daily arithmetic returns of assets versus the market. Second the expected asset return or drift, μ , is calculated by multiplying the estimated asset beta in the previous step by the equity risk premium (assumed to be 4.8%) and adding the safe rate. If calculated asset drift is negative, then the safe rate will be used.

Distance to and Probability of Default

Now we can directly enter the firm's most current estimated asset value and the TTM asset volatility and drift calculated from the preceding section into the distance to and probability of default equations (4) and (5).

Ranking and Mapping of Distance to Default to Financial Health Grades

In the final step, we rank the calculated distance to default and assign alphabetical financial health grades A through F based on the following percentile rankings:

Financial Health Grades:

Percentile Rank – Low to High	Cumulative Percentile	Grade
10%	10%	F
20%	30%	D
40%	70%	C
20%	90%	B
10%	100%	A

Future Enhancements

Although empirical data support the application of distance-to-default-based traditional structural models for financial services companies, future enhancements of this algorithm, particularly in regard to banking institutions, would include a distance-to-capital requirement to account for the regulatory framework that provides a higher degree of freedom than those available to nonfinancial companies.