Holdings-Based and Returns-Based Style Models

June 2003

Paul D. Kaplan, Ph.D., CFA
Director of Research
Morningstar, Inc.
225 West Wacker Drive
Chicago, IL 60606

The author thanks Robert Morioka at Morningstar for performing the holdings-based analysis and providing the data needed to perform the returns-based analysis. The author also thanks Julie Austin and Michele Gambera of Morningstar, Barton Waring of Barclays Global Investors, Laurence Siegel of the Ford Foundation, and James Knowles of York Hedge Fund Strategies for their helpful comments.

Introduction

Investment style is now the dominant principle used to classify, analyze, and deploy equity portfolios. Investment research firms classify equity funds for ratings and other purposes into categories based on investment style. Institutional investors, consultants, financial investors, and individuals use investment style as a criterion for selecting funds, either to achieve diversification or make style bets. In response to the emphasis that investors place on investment style, many equity mutual funds identify themselves as being of a certain style by using phrases such as "mid-cap growth" or "small company value" in their names.

With the growing emphasis on investment style came the need for style analysis tools. On the one hand, because portfolio managers do not always follow their stated style mandates (or even have stated style mandates), investors and their advisors need to be able to independently determine a portfolio's style. On the other hand, portfolio managers who are concerned about how investors and their advisors perceive their style need tools to verify that they are remaining true to their intended style.

It is now a generally accepted principle that a portfolio manager who follows a particular investment style should be evaluated against a passive benchmark that has the same style. This leads to the twofold problem of constructing style specific benchmarks and matching funds to the right benchmarks. Since few investment styles exactly match the construction rules of any single published index, it is often necessary to create custom benchmarks.[1] Style analysis can be used to create custom benchmarks in the form of fund-specific combinations or "portfolios" of indexes.

The same analysis can also be used to provide a more detailed description of investment style than is revealed by a fund category assignment. Rather than stating that a fund belongs in, say, the "large-cap growth" category, many equity style models assign a pair of numerical scores for size and value/growth orientation that can be plotted on an x-y grid.[2] The position of a fund's point on the grid makes distinctions such as "core growth" and "high growth" visually apparent. <u>If done accurately</u>, such plots are extremely useful in showing the distinctions between the investment styles of funds that fall into the same style category. This is why the ability to create such plots is a key feature of many commercially available style analysis software packages.

Inaccurate analysis can lead to extremely inaccurate conclusions. A misleading analysis that is easy to perform is worse than no analysis at all. Therefore, it is important for the users of style analysis to understand how the models work and be familiar with their limitations before putting them into practice.

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[1] Quantitative active managers who use a published style index as their starting point are an exception to this rule.

[2] Sharpe [1988] introduced this type of investment style grid.

There are two main approaches to style analysis: holdings-based and returns-based. There has been much debate between proponents of these two approaches. Most of the debate has focused on the relative accuracy of the two methods in describing a fund's allocation among asset classes or equity styles.[3] However, no previous study compares the style plot points generated by the two methods. This study fills this gap by (1) developing a method to display the style plot points generated by the two methods, and (2) comparing the style plot points generated by the two methods over a large set of U.S. equity funds. We highlight where the results are similar and where they significantly differ. Where there are significant differences, we explore some of the possible reasons. Users of style analysis should find this study helpful in determining which, if either, method is appropriate for their applications.

We start with an overview of holdings-based and returns-based style analysis in general and an overview of this study.

Overview of Style Analysis

Holdings-Based

Holdings-based style analysis is a "bottom-up" approach in which the characteristics of a fund over a period of time are derived from the characteristics of the securities it contains at various points in time over the period. The choice of characteristics depends on the purpose of the analysis. If the purpose is to create a customized benchmark consisting of a portfolio of indexes or to decompose the portfolio into a set of asset classes, the only security characteristic needed is index or asset class membership. If the purpose is to describe a portfolio in terms of a set of quantitative style characteristics such as size and value/growth orientation, the prescribed characteristics of each security need to be calculated and then aggregated to the portfolio level.

Holdings-based style analysis requires two sets of data. First, we need a security database that contains the characteristics of each security in the investable universe of the funds being analyzed. Second, we need a record of the security holdings of each fund being analyzed. Each database must contain the requisite data for each time period being studied.

The databases needed to perform holdings-based style analysis are expensive to obtain and keep up to date. Because of this, there are only a handful of investment research firms that have the needed datasets and perform holdings-based style analysis.

Returns-Based

Sharpe [1988, 1992] introduced a low cost alternative to holdings-based style analysis, namely, returns-based style analysis. Sharpe's approach is to regress a fund's historical returns against the returns of a set of passively constructed reference portfolios, each

[3] See for example Rekenthaler, Gambera, and Carlson [2002] and Buetow, Johnson, and Runkle [2000].

reference portfolio representing an asset class or an investment style. The coefficients on the reference portfolio returns are constrained to be nonnegative and sum to one so that they represent a long-only portfolio of passive investments. This portfolio serves as the fund's custom benchmark.

Sharpe's model made style analysis readily available to anyone who could obtain historical returns data on the portfolio being analyzed and on passive indexes. Due to the importance of style analysis and relative inexpensiveness of returns data, Sharpe's model quickly became popular among institutional investors and consultants. Several firms developed software packages for both the institutional and advisor markets to perform returns-based style analysis.

Most of these software packages create plots of equity style characteristics of funds. To do this, they first assign a point in x-y space to each reference portfolio that represents a specific equity style, such as large-cap value. They then generate a plot point for the fund in question by taking a weighted average of the plot points of the reference portfolios, using the results of returns-based style analysis for the weights.

Overview of this Study

To generate style plot points from holdings-based analysis, we use the style model that Morningstar introduced in 2002 to analyze stocks and equity funds and to construct equity style indexes.[4] This model assigns an x-y coordinate pair to most U.S. stocks each month. The x-coordinate represents the value/growth orientation and the y-coordinate represents size. The coordinate pairs of the stocks in a portfolio can be rolled up into the portfolio's "centroid" by taking the asset-weighted average of the stock locations.[5] A centroid represents the overall investment style of the portfolio. The portfolio centroids of a fund from different points in time can be averaged to measure the fund's long-term style.

Using the Morningstar style model, we divide the stock universe into style specific portfolios to construct the reference portfolios for our returns-based style model. The style plot points of the reference portfolios are the average portfolio centroids derived from their holdings. Using the same underlying model to construct the reference portfolios as we use for our holdings-based analysis should increase the likelihood that the two methods will produce similar results.

For our returns-based model, we use a standard regression model that does not constrain the coefficients to be nonnegative. We argue that such constraints would preclude the

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4 For a description of this model and some of its application, see Kaplan, Knowles, and Phillips [2003].

5 An object's centroid, or center of gravity, is the average location of its weight. We apply the term "centroid" to the average location of a portfolio's asset weights on a two-dimensional grid that represents investment style. Sharpe [1988] uses the term "center of gravity" in a similar fashion.

model from ever generating the appropriate style plot points for funds that have strong value or growth orientation, such as "deep value" or "high growth," or an extreme size bias such as "giant-cap" or "micro-cap."[6] We also argue that the model proposed by Fama and French [1993, 1995, 1996], which has become popular with academic researchers and some institutional practitioners, contains overly restrictive constraints when implemented as a model of fund style.[7]

We apply the estimated regression coefficients to the centroids of the reference portfolios to estimate style plot points that ought to be comparable to the time-averaged centroids obtained through holdings-based style analysis. We also construct statistical confidence regions around the returns-based estimated centroids.

We first compare the results of the two methods for category averages. We then perform our analysis on 1,909 distinct U.S. equity mutual funds from the nine U.S. diversified equity Morningstar Categories ("Large Value," Large Blend," etc.). We find that the degree of similarity of the results of the two methods varies widely across funds. In some cases, the dissimilarity that we see is to be expected, but in other cases it is quite surprising. To get an overall picture, we look at the overall correlation between the centroid coordinates generated by the two methods.

We then look into possible causes for substantial differences between the results of the two methods. First, we see whether the degree of similarity is correlated with the goodness-of-fit of the regression. We then consider if variation across time of style weights could be problematic for returns-based style analysis by constructing a simple simulation.

The Morningstar Equity Style Model

In 2002, Morningstar changed the model that it uses to classify U.S. equity funds by style. In the new model, stocks are first classified by their market capitalizations as being large-cap (the top 70% of the market), mid-cap (the next 20%), small-cap (the next 7%), or micro-cap (the bottom 3%). Stocks within each of these bands are scored on a relative basis on up to five value factors and five growth factors.[8] An overall value score is calculated as a weighted average of the value scores and an overall growth score is calculated as a weighted average of the growth scores. Exhibit 1 lists the 10 factors and their weights in the overall value and growth scores.

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6 Barton Waring of Barclays Global Investors pointed this out to me and recommended using unconstrained regression.

7 Although the Fama-French model was originally developed as an asset-pricing model, it has become popular as a form of returns-based style analysis. For example, Brown and Harlow [2002] use a variation of the Fama-French model as a style model.

8 Micro-cap stocks are scored using the scales of the small-cap band.

Value/growth orientation is measured using the overall growth score minus the overall value score of each stock. Exhibit 2 illustrates how the values of the 10 factors are used to measure the valuation/growth orientation of a stock using Nike as an example.

The x-coordinate on the style grid of each stock is derived from the value/growth orientation measure. The y-coordinate is derived from the logarithm of the stock's market capitalization.[9] Exhibit 3 illustrates how the data on Nike shown in Exhibit 2 translates into the x- and y-coordinates for Nike.

By plotting the style coordinates of all of the stocks held by a fund, we can generate a detailed graphical representation of the fund's style at a point in time. Exhibit 4 shows such a plot for the Lord Abbett Large-Cap Research fund as of December 31, 2002. The point inside the small circle is the portfolio centroid. It is calculated by taking the asset-weighted average of the x- and y-coordinates of the stocks. The large ellipse is what Morningstar calls the fund's Ownership Zone[SM]. The Ownership Zone contains the 75% of the fund's stock assets that are closest to the portfolio centroid. While the centroid indicates a fund's overall style, the Ownership Zone shows how concentrated a fund is around its overall style and thus provides additional insight into a fund's style management.

The grid in Exhibit 4 is composed of five rows of five squares each. Along the horizontal axis, the central range is where the centroids of blend funds lie. The two ranges to the left represent "core" and "deep" degrees of value. Similarly, the two ranges to the right represent "core" and "high" degrees of growth. On the grid, the widths of the five ranges are depicted as being equal, even through the corresponding ranges of the values of the x-coordinate are unequal. Hence when we depict style coordinates on the 25-square grid, we rescale the value of the x-coordinate using a piecewise linear function.

We perform a similar rescaling of the value of the y-coordinate. The center, below center, and bottom ranges along the vertical axis represent the ranges of the logarithm of market capitalization for mid-cap, small-cap, and micro-cap stocks respectively. The top range represents the range for "giant-cap" stocks, which we define as the stocks that make up the top 40% of the market capitalization of the U.S. equity market. The range between giant-cap and mid-cap stocks represents the remaining large-cap stocks. Since the corresponding ranges of the values of the y-coordinate are unequal, we rescale the value of the y-coordinate using a piecewise linear function when depicting style coordinates on the 25-square grid.[10]

Since portfolio centroids are averages of stock coordinate pairs, they tend to fall closer to the center square on the grid than the coordinate pairs of individual stocks. Hence the centroids of most funds fall within the nine inner squares of the 25-square grid. This is

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9 See Morningstar [2002] for the algorithm that Morningstar uses to generate the x- and y-coordinates from the values of the 10 factors and the market capitalization of a stock.

10 See Morningstar [2003] for specifications of the rescaling functions.

why these nine squares are outlined in Exhibit 4 and labeled the "fund grid." Outlining these nine-squares gives the central part of the grid an appearance that is similar to the Morningstar Style Box™. We use the 25-square grid with piecewise linear scaling to depict all of our style analysis results in this paper.

Morningstar assigns a diversified U.S. equity fund into one of nine categories (large-cap value, large-cap blend, etc.[11]) largely (but not exclusively) on the basis of the three-year average of the fund's portfolio centroids. The most recent 36 months are divided into three 12-month periods. The x- and y-coordinates of all portfolios for which Morningstar has data for the fund are averaged using equal weights. The resulting average coordinates for the three 12-month periods are averaged using equal weights to obtain the three-year average centroid. In this way, 12-month periods that contain unequal numbers of portfolio reports are weighted equally. Exhibit 5 illustrates the procedure using data for the Lord Abbett Large-Cap Research fund over the period January 2000 through December 2002. We use these three-year average centroids for holdings-based style analysis.

Returns-Based Style Analysis

For returns-based style analysis, we use the following model:

$$R_{Ft} = \alpha + \beta_C R_{Ct} + \beta_{LV} R_{LVt} + \beta_{LG} R_{LGt} + \beta_{SV} R_{SVt} + \beta_{SG} R_{SGt} + \varepsilon_t \qquad (1)$$

where

R_{Ft} = total return on the fund in month t
R_{Ct} = total return on cash in month t
R_{LVt} = total return on the large-cap value reference portfolio in month t
R_{LGt} = total return on the large-cap growth reference portfolio in month t
R_{SVt} = total return on the small-cap value reference portfolio in month t
R_{SGt} = total return on the small-cap growth reference portfolio in month t
α = intercept
β_C = coefficient on cash
β_P = coefficient on portfolio P (P = LV, LG, SV, SG)
ε_t = error term in month t

Using four "corner" equity style reference portfolios as we do here is typical in returns-based style analysis. Doing so allows us to span both dimensions of the style grid independently with the fewest possible number of reference portfolios.

In standard regression analysis, each beta coefficient measures the sensitivity of the dependent variable to its corresponding independent variable. However, in returns-based

[11] These are nine out of the 58 categories that Morningstar uses to classify funds. The other 49 categories are for specialty equity, international equity, asset allocation, and fixed income funds.

style analysis, the betas are also interpretable as portfolio weights.[12] Hence, we constrain them to sum to one:

$$\beta_C + \beta_{LV} + \beta_{LG} + \beta_{SV} + \beta_{SG} = 1 \qquad (2)$$

Using equation (2) to substitute β_C out of equation (1) and rearranging terms, we obtain the following regression model:

$$r_{Ft} = \alpha + \beta_{LV} r_{LVt} + \beta_{LG} r_{LGt} + \beta_{SV} r_{SVt} + \beta_{SG} r_{SGt} + \varepsilon_t \qquad (3)$$

where

r_{Pt} = total return on portfolio P in month t minus return on cash (i.e., excess return) (P = LV, LG, SV, SG)

The coefficients in equation (3) can be estimated using ordinary least squares. Also standard statistical techniques can be used to construct confidence regions on functions of the coefficients, such as estimated portfolio centroids.

We construct the equity style reference portfolios by dividing the 25-square grid into the four large squares shown in Exhibit 6, and forming float-adjusted capitalization-weighted portfolios of the stocks that fall into these large squares each month. Exhibit 6 also shows the three-year average centroids of the four equity style reference portfolios. Using the same underlying model to construct the reference portfolios in the returns-based analysis as we use for our holdings-based analysis should give us the best chance of the two methods producing similar results.

We use the beta estimates to form returns-based centroid estimates that are comparable to the holdings-based estimates. To do this, we define the estimated total allocation to equity as

$$\hat{\beta}_Q = \hat{\beta}_{LV} + \hat{\beta}_{LG} + \hat{\beta}_{SV} + \hat{\beta}_{SG} \qquad (4)$$

where

$\hat{\beta}_P$ = the estimate of β_P (P = LV, LG, SV, SG).

The estimated centroid coordinates are:

$$x_E = \frac{\hat{\beta}_{LV} x_{LV} + \hat{\beta}_{LG} x_{LG} + \hat{\beta}_{SV} x_{SV} + \hat{\beta}_{SG} x_{SG}}{\hat{\beta}_Q} \qquad (5)$$

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12 Sharpe [1988, 1992] at first refers to the coefficients as "sensitivities" and then changes the term to "weights."

$$y_E = \frac{\hat{\beta}_{LV} y_{LV} + \hat{\beta}_{LG} y_{LG} + \hat{\beta}_{SV} y_{SV} + \hat{\beta}_{SG} y_{SG}}{\hat{\beta}_Q} \qquad (6)$$

where

x_P = the x-coordinate of the average centroid of reference portfolio P
y_P = the y-coordinate of the average centroid of reference portfolio P
(P = LV, LG, SV, SG)

Unconstrained Regression vs. the Sharpe Model
Our returns-based style model differs from Sharpe [1988, 1992] in two respects.[13] Firstly, like Hardy [2003], we only include reference portfolios for cash and U.S. equities. Since we only analyze U.S. equity funds, any apparent allocation to another asset class that would result from Sharpe's model would likely be spurious. We include cash so that we can get the overall fund beta right; in other words, so we can model a fund as taking a levered or delevered position in the equity market, as in the Capital Asset Pricing Model (CAPM) suggests.

The second difference between our returns-based model and Sharpe's is that we do not constrain any of the betas to be nonnegative[14]. Constraining the beta on cash to be nonnegative would allow for delevered equity portfolios, but would exclude the possibility of leverage. This would be like estimating the beta of a portfolio using the CAPM, but only allowing the beta to be between zero and one.

Constraining the style betas (β_{LV}, etc.) to be nonnegative would prevent the model from identifying funds as having more extreme styles such as giant-cap or high growth. This can be seen in Exhibit 7 where we indicate that the Sharpe model would limit estimated centroids to fall within the quadrangle formed by the centroids of the four equity reference portfolios. Many funds could not be adequately modeled if we imposed Sharpe's non-negativity constraints.

Sharpe [1988] noted that his model could be estimated using ordinary least squares if the non-negativity constraints were dropped, but chose to impose them because portfolios of indexes with negative weights would be impractical as actual benchmarks for long-only investors. However, imposing the constraints does not necessarily produce the best model of the manager. For example deep value might best be represented by leveraged positions in the value indexes and short positions in the growth indexes. Thus the unconstrained version of the style regression – which Sharpe [1992] discusses – conveys information

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13 Our inclusion of the intercept term (α) and Sharpe's apparent exclusion of it does not actually constitute a difference. As Becker [2003] explains in detail, because Sharpe's method is to minimize the variance of the error term, not the sum of squared errors, his model has the intercept term implicitly.

14 For a detailed discussion of the relationships between the completely unconstrained regression model implied by equation (1), the regression model that results from imposing the constraint in equation (2), and Sharpe's model, see DeRoon, Nijman and TerHorst [2000].

about the manager that is not in the constrained model. This information is manifested on the style grid when the estimated centroid falls outside of the quadrangle in Exhibit 7. Also, if the constraints are binding, the goodness-of-fit is necessarily better with the unconstrained regression. Furthermore, if a coefficient is constrained to be zero, the corresponding index return series could be correlated with the error term. This is problematic because the error is supposed to represent security selection effects, which in principle ought to be statistically independent of style effects. For all of these reasons, we adopt the unconstrained approach to style analysis in this paper.

Unconstrained Regression vs. the Fama-French Model

The model introduced by Fama and French [1993, 1995, 1996] is often used in academic research as a form of returns-based style analysis for U.S. equity portfolios. Using our reference portfolios, the Fama-French model can be implemented as follows:[15]

$$r_{Ft} = \alpha + \beta_Q r_{Mt} + \gamma_x \left(r_{LVt} + r_{SVt} - r_{LGt} - r_{SGt} \right) + \gamma_y \left(r_{SVt} + r_{SGt} - r_{LVt} - r_{LGt} \right) + \varepsilon_t \tag{7}$$

where

r_{Mt} = total return on the equity market portfolio minus return on cash in month t
γ_x = a parameter that measures the fund's value/growth orientation
γ_y = a parameter that measures the fund's size orientation

and all other symbols are as they are defined previously.

Suppose that the equity market portfolio were a fixed-weight combination of our four equity style reference portfolios.[16] We could write

$$r_{Mt} = w_{LV} r_{LVt} + w_{LG} r_{LGt} + w_{SV} r_{SVt} + w_{SG} r_{SGt} \tag{8}$$

where

w_P = the market weight of reference portfolio P.

By definition,

$$w_{LV} + w_{LG} + w_{SV} + w_{SG} = 1 \tag{9}$$

15 Fama and French decompose the market into six portfolios by including a "neutral" category between value (high book-to-market) and growth (low book-to-market.) They include the neutral portfolios in the size factor, but exclude them from the value/growth orientation factor. Since we do carve out a neutral territory like this, we do not do this in our interpretation of their model.

16 Regressing the excess return on the float-adjusted capitalization-weighted composite of the four reference portfolios on the excess returns of four reference portfolios over the 36-month period January 2000 – December 2002 yields an R-squared value of 99.8%. So the fixed-weight assumption is a reasonable one for our purpose here.

From equations (3), (8), and (9), it follows that our implementation of the Fama-French model is equivalent to our returns-based style model in equation (3) with the constraint that

$$\beta_{LV} + \beta_{SG} = \left(w_{LV} + w_{SG}\right)\beta_Q \tag{10}$$

or equivalently[17]

$$\beta_{LG} + \beta_{SV} = \left(w_{LG} + w_{SV}\right)\beta_Q \tag{11}$$

While this constraint does not limit the range of estimated centroids, it does place a severe constraint on the custom benchmarks that the model can generate from combinations of the reference portfolios. In Exhibit 8, the diamond-shaped area shows the centroids of custom benchmarks that can be obtained from long-only combinations of the reference portfolios under the constraint of the Fama-French model. The fact that not even a 100% allocation in any single reference portfolio can be obtained demonstrates the severity of this constraint. Hence, the Fama-French model is not a particularly useful way of doing returns-based style analysis. An unconstrained regression model, such as equation (3), is the most flexible returns-based approach to estimating style centroids and creating custom benchmarks.

Data and Calculations

The data used in this study cover the period January 2000 through December 2002. All of the data were obtained from Morningstar's internal databases. From the stock database we obtained:

- The monthly values of x- and y-coordinates of each stock.
- Where available, the float-adjusted market capitalization of each stock. Where a float-adjustment is not available, the unadjusted market capitalization.
- Month-end share prices and total monthly dividend payments.

[17] The constraint follows from the mapping of the three coefficients of the Fama-French model into the style coefficients as follows:

$\beta_{LV} = \beta_Q w_{LV} + \gamma_x - \gamma_y$ $\quad\quad$ $\beta_{LG} = \beta_Q w_{LG} - \gamma_x - \gamma_y$

$\beta_{SV} = \beta_Q w_{SV} + \gamma_x + \gamma_y$ $\quad\quad$ $\beta_{SG} = \beta_Q w_{SG} - \gamma_x + \gamma_y$

From the fund database we obtained the following data on diversified U.S. equity funds:

- The category of each fund as of February 2003.

- Month-end security identifiers and dollars invested in each security of each fund, to the extent that fund companies reported them to Morningstar over the 2000-2002 period.[18]

- Monthly total returns on each share class of each fund that has a complete return history over the 2000-2002 period.

We found the requisite data on 1,909 "distinct" funds (that is, counting multiple share classes as one fund). Exhibit 9 shows the number of distinct funds in each of the nine Morningstar Categories of U.S. diversified equity funds.

From the x- and y-coordinates of the stocks and the holdings data on the funds, we calculate the three-year average centroids of the funds. The category average centroids are the simple averages of the three-year average centroids of all funds within each category.

From the share price and dividend data, we calculate the monthly total return of each stock. Based on each stock's x- and y-coordinates each month, we place each stock each month into one of the four reference portfolios. For each month we calculate the float-adjusted capitalization weighted total return and centroid. We average the monthly centroid coordinates to form the reference portfolio centroids shown in Exhibit 6.

For funds with multiple share classes, we calculate the simple average of the total returns of the various share classes for each month to obtain a single time-series of total returns for each distinct fund. Category average returns are the simple averages of the resulting returns on all distinct funds within each category.

We subtract the return on cash[19] each month from the total returns on the funds, the category averages, and the reference portfolios, and then estimate the parameters of equation (3) for each fund and category average using ordinary least squares regression. Using equations (4), (5), and (6) we calculate the returns-based centroid estimate of each fund and category average. Using the method described in the Appendix, we calculate a 95% confidence region around each estimated centroid.

[18] Nearly all funds report portfolio holdings semiannually to Morningstar since they have to report this information to their shareholders anyway in accordance with SEC regulations. Most fund companies, including 71 out of the largest 75, go beyond this, reporting portfolio holdings to Morningstar quarterly or monthly.

[19] Returns on cash are calculated by Morningstar from yields on 90-day Treasury bills.

Results

Category Averages

We first compare the results of holdings-based and returns-based centroid estimates for category averages. Exhibit 10 shows the R-squared values for each of the nine category average regressions. They are all quite high, the lowest being 94.8%.

Exhibit 11 shows the estimated centroids and confidence regions for the nine category averages. In all cases, the estimated centroids fall in the right general area of the grid. However, the returns-based estimates for mid-cap growth and small-cap growth fall into the extreme area of the grid. This happens because in these cases, the estimated coefficient on the large-cap value reference portfolio is negative in sign and large in magnitude[20]. This raises questions about the reliability of the returns-based method.

The confidence regions shown in Exhibit 11 are much larger for mid-cap growth, small-cap blend, and small-cap growth than they are for the other category averages, even though the R-squared values are not much lower.[21] This is because in these regressions, the volatility of the error term, as measured by the standard error of regression, is significantly larger than in the other regressions. Also, the total estimated allocation to equity – $\hat{\beta}_Q$ as defined by equation (4) – is lower in these regressions than in the other regressions (81-84% as opposed to about 100%)[22]. This lowers the statistical significance of the point estimates and hence enlarges the confidence regions.[23]

Individual Funds

At the individual fund level, there are many cases in which the results of holdings-based analysis and returns-based analysis can be quite similar. As shown in Exhibit 12, this is the case for some very well known funds.[24] However, there can also be substantial differences as shown in Exhibit 13 .[25]

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[20] -19% and -32% respectively.

[21] In ordinary x-y space, the confidence regions would be perfectly elliptical. However, because of the piecewise linear rescaling that we use when plotting style coordinates on the 25-square grid, the ellipses are distorted when they cross two or more squares.

[22] Standard error and beta estimates on category average regressions are available from the author.

[23] See equation (A.13) in the appendix.

[24] Ben Dor, Jagannathan, and Meier [2003] also present returns-based results for Vanguard Growth and Income as well as four other funds to demonstrate the value of returns-based style analysis. For all five of their sample funds, the results of returns-based and holdings-based analysis are similar in our analysis.

[25] The regression for Legg Mason Value (Exhibit 12) and Van Kampen American Value (Exhibit 13) have similar R-squared values and standard errors of regression. The difference in the sizes of their respective confidence regions is due to a large difference in their respective estimated allocations to equity: 129% for Legg Mason vs. 64% for Van Kampen.

Returns-based style analysis can be useful for funds that get their exposure to an asset class by taking long positions in index futures rather than by holding the underlying securities outright. A good example of this is the PIMCO StocksPlus fund. This fund takes long positions in S&P 500 futures contracts, which it fully collateralizes with fixed income investments. As shown in Exhibit 14, returns-based style analysis correctly models the portfolio as having nearly the same style characteristics as the S&P 500.

However, returns-based analysis does not work well with all non-conventional investment practices. For example, the manager of Needham Growth fund regularly took short positions in high growth stocks and growth exchange traded funds over our period of study.[26] This should have given the fund's return pattern a more moderate growth tilt than would be evident from its direct holdings. We would expect the results of returns-based style analysis to reflect this. However, as shown in Exhibit 15, the opposite seems to occur. As we discuss later, this could be due to a poor goodness-of-fit of the regression,[27] or the time variation in style exposures, or a combination of both.

Exhibit 16 presents the averages of the R-squared values from the individual fund regressions by category. For large-cap funds, average R-squared value is over 90%. For mid-cap and small-cap funds, it is a bit lower at about 85%. This would suggest that the style regressions do a reasonably good job of describing the return patterns – although not necessarily the actual style exposures – of funds.

However, as we saw with some particular funds, high values of R-squared may or may not correspond with good estimates of style characteristics. Exhibit 17 shows that for large-cap value funds, holdings-based and returns-based estimated centroids both are concentrated in the large-cap value area of the style grid.[28] However, large-cap value is the only one of the nine diversified U.S. equity fund categories for which this holds. For example, as Exhibit 18 shows, the picture is very different for mid-cap blend funds. While the holdings-based centroid estimates are concentrated in the mid-cap blend square, the returns-based estimates are scattered across the grid. This occurs even though the rescaled[29] estimated coordinates between the two methods are more correlated for mid-cap blend funds than they are for large-cap value funds. Exhibit 19 shows the results for small-cap growth funds where the correlation between the rescaled y-coordinates from the two methods breaks down completely.

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26 See Sweeney [2003] for a description of then fund manager Peter Trapp's investment practices. Trapp left Needham in 2003 to enter the hedge-fund business.

27 A relatively low R-squared value and a very high standard error of regression suggest that a returns-based model might give poor representation of this fund.

28 Since Morningstar used the three-year average holdings-based centroid as the main criterion for classifying diversified U.S. equity funds, the concentrations seen on the holdings-based side of Exhibits 17, 18, and 19 are mainly by construction.

29 I.e., rescaled as described in Morningstar [2003].

Exhibits 20 and 21 show that while overall, there is a high correlation between holdings-based and returns-based rescaled centroid coordinates, the relationship does not hold up for many funds, especially for the y-coordinate of mid-cap and small-cap funds.

Reasons Why Returns-Based Style Analysis Might Break Down

Poor Goodness-of-Fit

If the style regression for a fund is a poor statistical model of the data, we would not expect it to provide a good representation of the fund's style. Hence we hypothesize that there is a systematic relationship between the differences in results between returns-based analysis and holdings-based style analysis and the statistical precision of the style regressions. To test this hypothesis, we calculate the difference in results for each and measure the correlation between the differences and the two measures of goodness-of-fit: R-squared and standard error of regression. We use a Euclidean measure of the difference in results, namely

$$ED = \sqrt{\left(\hat{x}_H - \hat{x}_E\right)^2 + \left(\hat{y}_H - \hat{y}_E\right)^2} \qquad (12)$$

where

$\hat{x}_H$ = the rescaled x-coordinate of the holdings-based estimated centroid
$\hat{x}_E$ = the rescaled x-coordinate of the returns-based estimated centroid
$\hat{y}_H$ = the rescaled y-coordinate of the holdings-based estimated centroid
$\hat{y}_E$ = the rescaled x-coordinate of the returns-based estimated centroid

Over the entire sample of 1,909 distinct funds, we find that the correlation between ED and R-squared is –52% and between ED and the standard error of regression is 63%. Both of these correlations are highly statistically significant for a sample of this size. Hence the goodness-of-fit of the regression is significantly related to differences in the results of the two models, but it is not the sole factor.

Style Inconsistency

As Elton and Gruber [1991] discuss, the appropriate benchmark for an active manager is the portfolio that he or she would hold in the absence of any information or insights about the future performance of the securities in his or her investment universe. Any information that the active manager receives should result in the appropriate overweighting and underweighting of securities relative to the benchmark. An active manager is successful if he or she is generating positive correlations between security weights and subsequent security returns.

If a manager's benchmark has a significant representation of more than one investment style and if the manager receives and acts on information about the future relative performance of those styles, the manager's style mix should deviate from the benchmark's over time, resulting in style inconsistency.[30]

To see what implications such style inconsistency might have for returns-based style analysis, we construct a synthetic series of fund returns in which the manager picks one of four mixes of the reference portfolios. We assume that the manager has perfect foresight at the end of each month as to which of the four reference portfolio will have the best performance in the following month and chooses the mix with the highest allocation to the best performing reference portfolio. The mixes are constructed so that the average allocation is 25% in each of the four reference portfolios. Exhibit 22 shows the four mixes and how they are constructed.

Exhibit 23 presents the results of the style regression for the synthetic portfolio. At 95.3%, the R-squared value would indicate a very good fit. The other regression statistics confirm this. Yet, the estimated coefficients are quite different than the actual average allocations of 25% to each reference portfolio. From this we conclude that the results of returns-based style analysis can be quite misleading if there is a significant amount of style inconsistency over the period of study.[31] The extent to which equity funds are style inconsistent[32] and the actual effects of style inconsistency on the results of returns-based style analysis are empirical questions that need further investigation.

Summary and Conclusions

Holdings-based and returns-based models are both used to describe investment style. In this study, we present a framework that allows us to do a systematic comparison of the results of the two methods and apply it to a large set of U.S. equity mutual funds.

For holdings-based style analysis, we use Morningstar's new 10 factor style model. The Morningstar model allows us to classify stocks, calculate style centroids and ownership zones for funds, and create reference portfolios for returns-based style analysis.

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30 As described here, style inconsistency should result in superior performance. Brown and Harlow [2002] claim to show that style inconsistency has been detrimental to performance. However, they use the R-squared from a Fama-French type model and tracking error against published style benchmarks as measures of style consistency rather than any measure based on actual style exposures. As we show below, goodness-of-fit statistics can be poor indicators of style consistency.

31 Practitioners of returns-based style analysis often use rolling overlapping periods to estimate style drift. However, this approach does not address the estimation problems caused by style inconsistency shown here. A more promising approach would be to embed the time variation of the style weights directly into the model as Spiegel, Mamaysky, and Zhang [2003] do in a single factor model.

32 A similar problem occurs if the performance effects of a fund's active security selection are correlated with the returns on the style reference portfolios.

For returns-based style analysis, we use an unconstrained linear regression model. We argue that the popular Sharpe and Fama-French returns-based models place severe and unrealistic constraints on reference portfolio weights. Our unconstrained model allows us to estimate style centroids and confidence regions in a straightforward manner.

We find that high R-squared values do not necessarily mean that returns-based style centroid estimates are reliable. Even when modeling portfolios of funds that all belong to the same style category, we find that the returns-based method can give extreme results with large confidence intervals, although the R-squared values are high.

We find that holdings-based and returns-based results are similar for many funds but differ substantially for others. Hence, holdings-based and returns-based analysis could lead to very different style classifications for many funds. We explore whether goodness-of-fit statistics for the style regressions can systematically explain the extent of the differences. We find that they are significantly related to differences in the results of the two models, but the goodness-of-fit of the style regression cannot be regarded as the sole factor leading to the differences. Other possible sources of differences include style inconsistency (which we demonstrate by a simulation) and correlation between selection and style effects. These possibilities require further investigation.

If a fund's portfolio is primarily composed of direct stock holdings, holdings-based analysis should be the primary means of assessing investment style. In the absence of such information, and <u>under the right conditions</u>, returns-based style analysis can be used to estimate investment style. Users of returns-based style analysis need to be aware of the conditions in which it produces inaccurate results. Users of all models need to keep in mind that quantitative techniques can complement, but never replace, qualitative knowledge of a fund's style and strategy.

Appendix: Confidence Regions for Estimate Style Centroids

Our unconstrained style regression equation is

$$r_{Ft} = \alpha + \beta_{LV} r_{LVt} + \beta_{LG} r_{LGt} + \beta_{SV} r_{SVt} + \beta_{SG} r_{SGt} + \varepsilon_t \tag{A.1}$$

Let

q_t = Excess return on fund in period t minus its time-series average
z_t = Vector of excess returns on the reference portfolios in period t minus the vector of their time-series averages
β = Vector of coefficients on the reference portfolios
σ^2 = $\mathrm{Var}[\varepsilon_t]$

We can rewrite equation (A.1) as follows:

$$q_t = \beta' z_t + \varepsilon_t \tag{A.2}$$

Let

T = Number of time-series observations
q = T-element vector of values of q_t
Z = T × 4 matrix of the values of z_t

The ordinary least squares estimators of β and σ^2 are

$$\hat{\beta} = \left(Z'Z\right)^{-1} Z'q \tag{A.3}$$

and

$$\hat{\sigma}^2 = \frac{\hat{\varepsilon}'\hat{\varepsilon}}{T-5} \tag{A.4}$$

where

$$\hat{\varepsilon} = q - Z\hat{\beta} \tag{A.5}$$

The estimated variance-covariance matrix of $\hat{\beta}$ is

$$\hat{\Sigma} = \hat{\sigma}^2 \left(Z'Z\right)^{-1} \tag{A.6}$$

Let

x_R = 4-element vector of x-coordinates of the centroids of the reference portfolios
y_R = 4-element vector of y-coordinates of the centroids of the reference portfolios

The returns-based estimate of the centroid coordinates are

$$x_E = \frac{x_R'\hat{\beta}}{\hat{\beta}_Q} \tag{A.7}$$

$$y_E = \frac{y_R'\hat{\beta}}{\hat{\beta}_Q} \tag{A.8}$$

where

$$\hat{\beta}_Q = \hat{\beta}_{LV} + \hat{\beta}_{LG} + \hat{\beta}_{SV} + \hat{\beta}_{SG} \tag{A.9}$$

The estimated asymptotic variance-covariance matrix for (x_R,y_R) is[33]

$$\hat{W} = \begin{bmatrix} \frac{\partial x_E}{\partial \hat{\beta}} & \frac{\partial y_E}{\partial \hat{\beta}} \end{bmatrix}' \hat{\Sigma} \begin{bmatrix} \frac{\partial x_E}{\partial \hat{\beta}} & \frac{\partial y_E}{\partial \hat{\beta}} \end{bmatrix} \tag{A.10}$$

From equations (A.7), (A.8), and (A.9), we have

$$\frac{\partial x_E}{\partial \hat{\beta}} = \frac{1}{\hat{\beta}_Q}\left(x_R - \iota x_E\right) \tag{A.11}$$

$$\frac{\partial y_E}{\partial \hat{\beta}} = \frac{1}{\hat{\beta}_Q}\left(y_R - \iota y_E\right) \tag{A.12}$$

ι being the vector of 4 ones.

From equations (A.6), (A.10), (A.11), and (A.12), we have

$$\hat{W} = \left(\frac{\hat{\sigma}}{\hat{\beta}}\right)^2 \left(x_R - \iota x_E \quad y_R - \iota y_E\right)'\left(Z'Z\right)^{-1}\left(x_R - \iota x_E \quad y_R - \iota y_E\right) \tag{A.13}$$

---

33 See Judge et al [1988], p. 542.

If x and y are the true values of the x- and y-coordinates, asymptotically, the quantity

$$\begin{pmatrix} x - x_E \\ y - y_E \end{pmatrix}' \hat{W}^{-1} \begin{pmatrix} x - x_E \\ y - y_E \end{pmatrix}$$

is a random variable with a chi-squared distribution with 2 degrees of freedom. Let

$\chi_2^2(p)$ = the critical value for a chi-squared distribution with 2 degrees of freedom for a 100p percent confidence region (5.99 for 95% confidence region)

The 100p percent confidence region around (x_E, y_E) is the set of coordinate pairs (x,y) that satisfies

$$\begin{pmatrix} x - x_E \\ y - y_E \end{pmatrix}' \hat{W}^{-1} \begin{pmatrix} x - x_E \\ y - y_E \end{pmatrix} \le \chi_2^2(p) \tag{A.14}$$

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Exhibit 1: Morningstar's Ten-Factor Style Model

Horizontal Axis: Style

Value Score Components and Weights		Growth Score Components and Weights	
Forward-looking measures	**50.0%**	**Forward-looking measures**	**50.0%**
► Price-to-projected earnings		► Projected earnings growth	
Historical-based measures	**50.0%**	**Historical-based measures**	**50.0%**
► Price-to-book	12.5%	► Book value growth	12.5%
► Price-to-sales	12.5%	► Sales growth	12.5%
► Price-to-cash flow	12.5%	► Cash flow growth	12.5%
► Dividend yield	12.5%	► Trailing earnings growth	12.5%

Vertical Axis: Market Capitalization

Top 70% of the market: Large Cap
Next 20%: Mid Cap
Next 10%: Small and Micro Cap

Exhibit 2: Style Factors for Nike

Value measures		Growth measures	
Price-to-projected earnings	18.5	Projected earnings growth	13.3%
Price-to-book	3.3	Book value growth	8.9%
Price-to-sales	1.3	Sales growth	5.0%
Price-to-cash flow	8.9	Cash flow growth	45.8%
Dividend yield	0.9 %	Trailing earnings growth	13.7%
Value Score (0-100)	47.5	Growth Score (0-100)	61.0

Style score : Growth score (61.0) – Value score (47.5) = 13.5

Market Cap: $13.6 billion

Exhibit 3: Style Coordinates for Nike

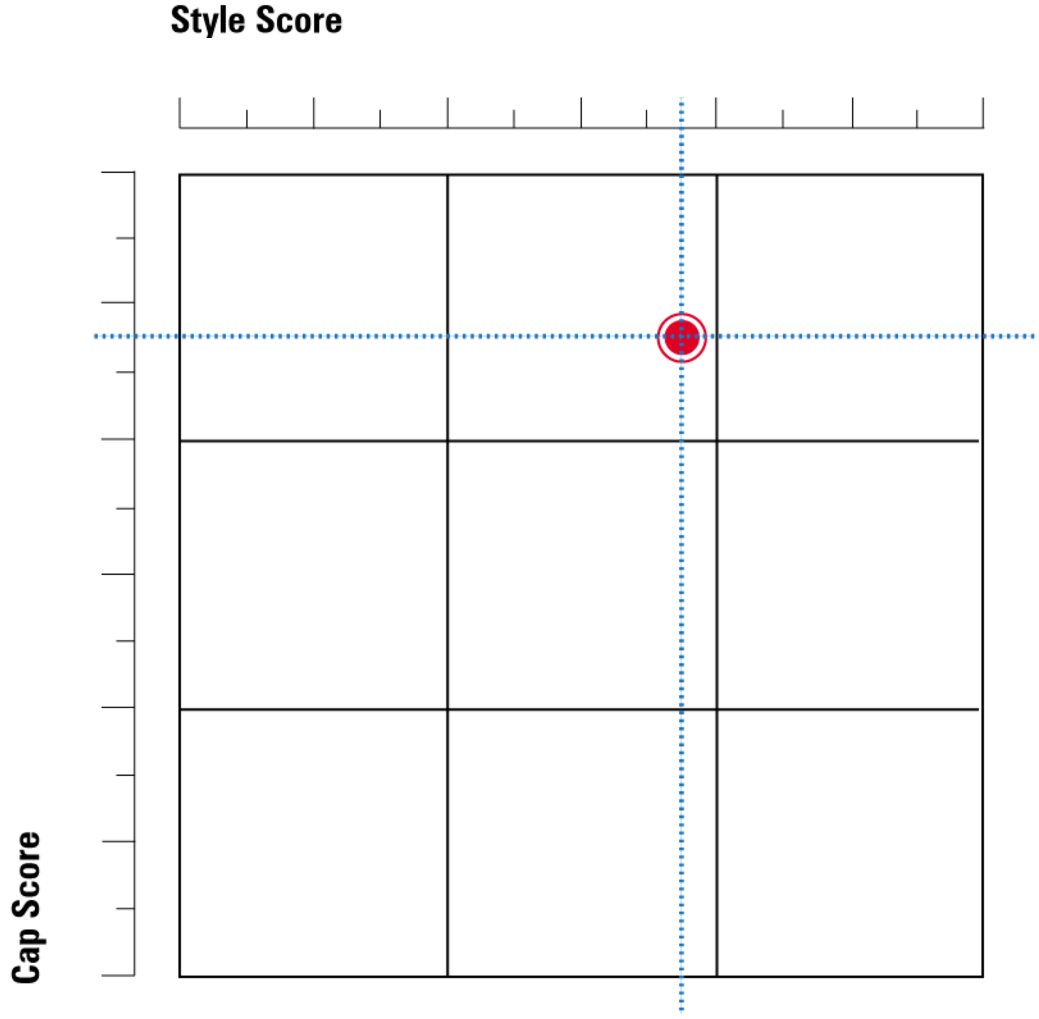


Exhibit 4: Ownership Zone for Lord Abbett Large-Cap Research

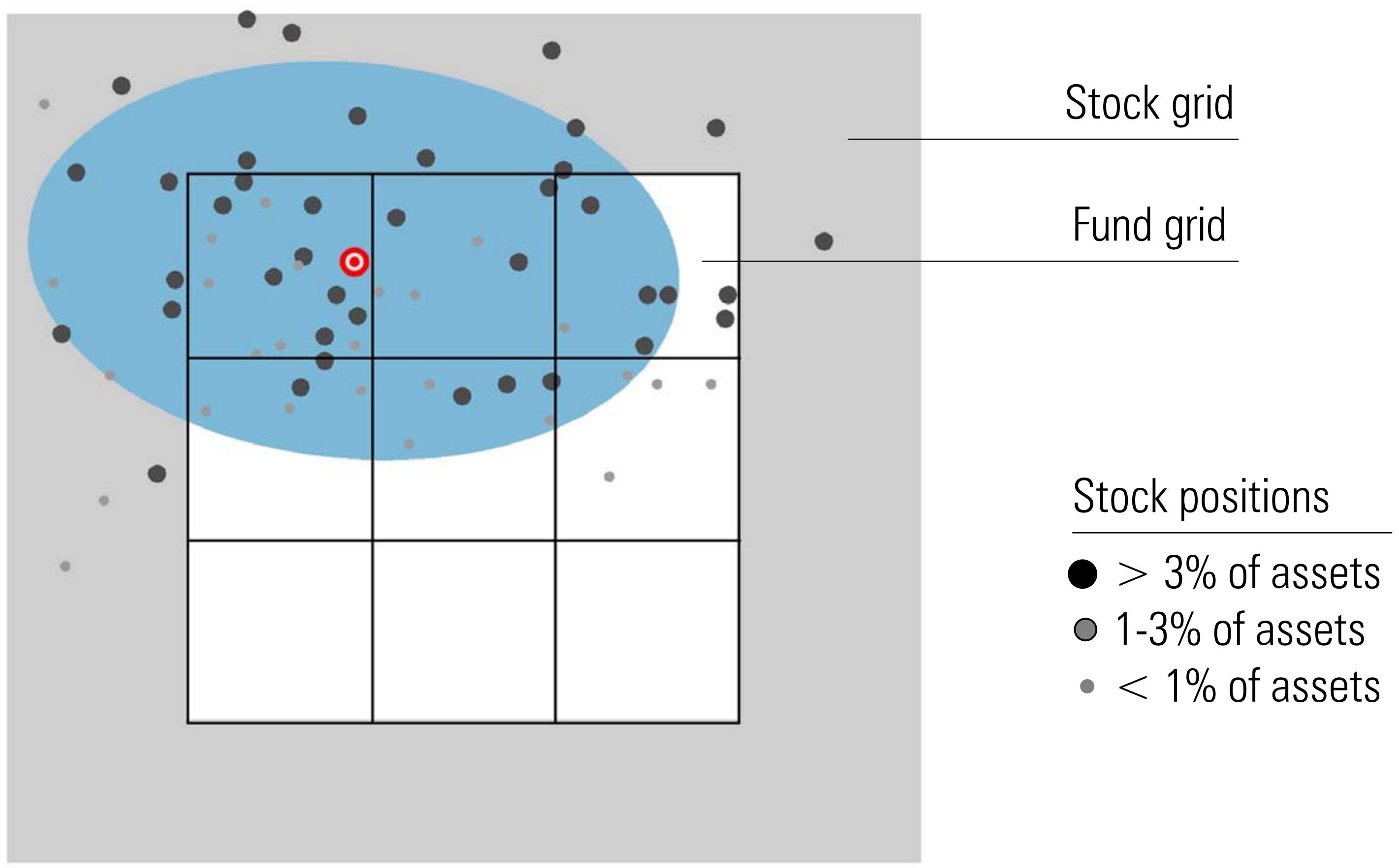
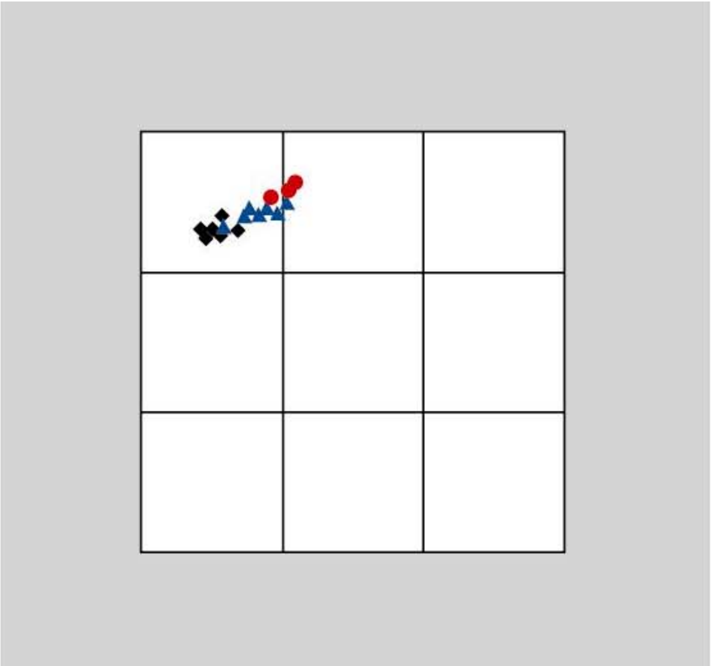
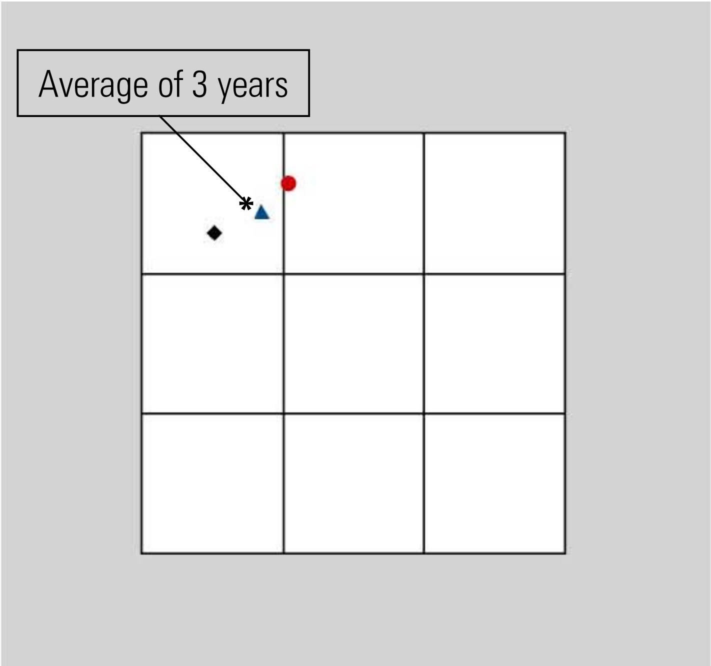


Exhibit 5: Centroids for Lord Abbett Large-Cap Research



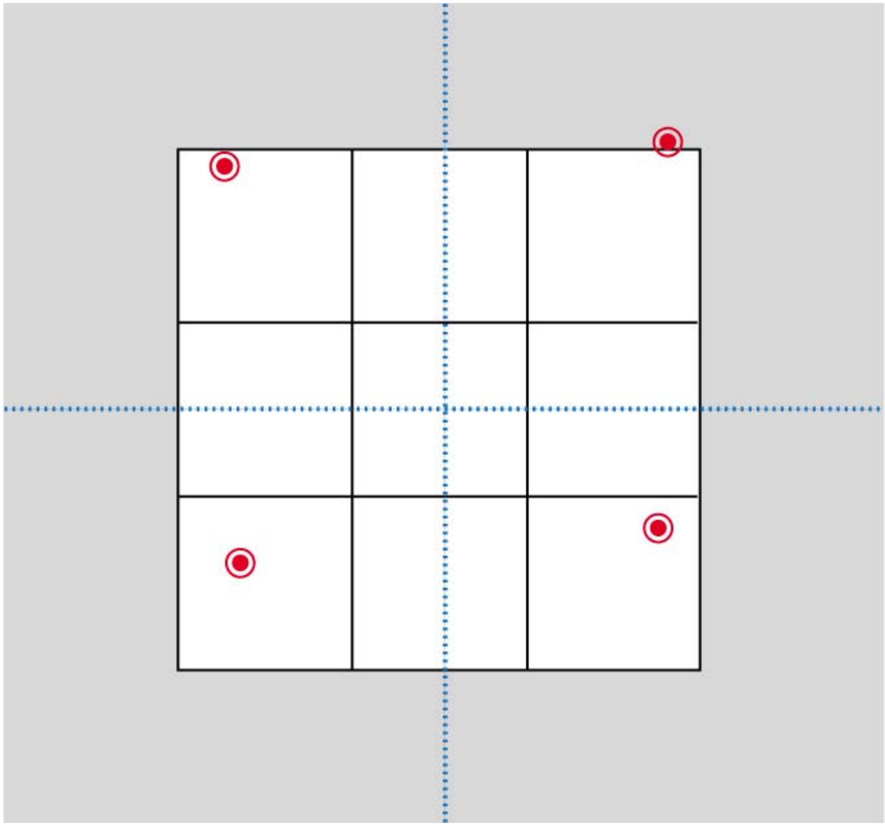
Month-End Portfolios



Annual Averages

◆ 2000
▲ 2001
● 2002

Exhibit 6: Reference Portfolio Centroids



Average over 2000-2002

Exhibit 7: Allowable Centroids in the Sharpe Model

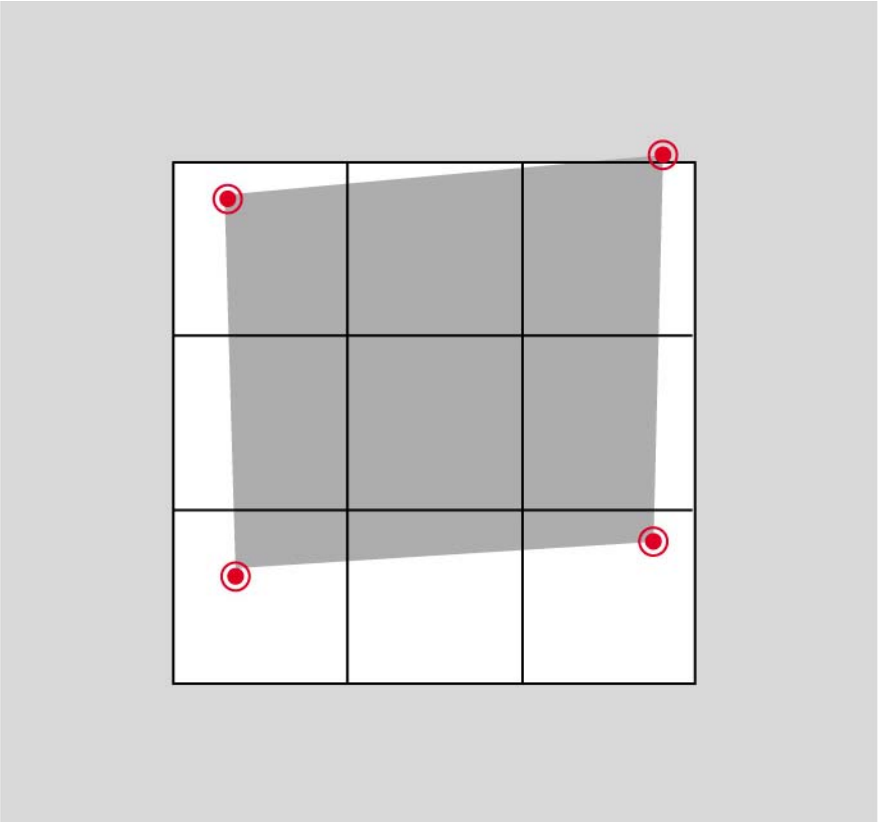


Exhibit 8: Style Grid in the Fama-French Model

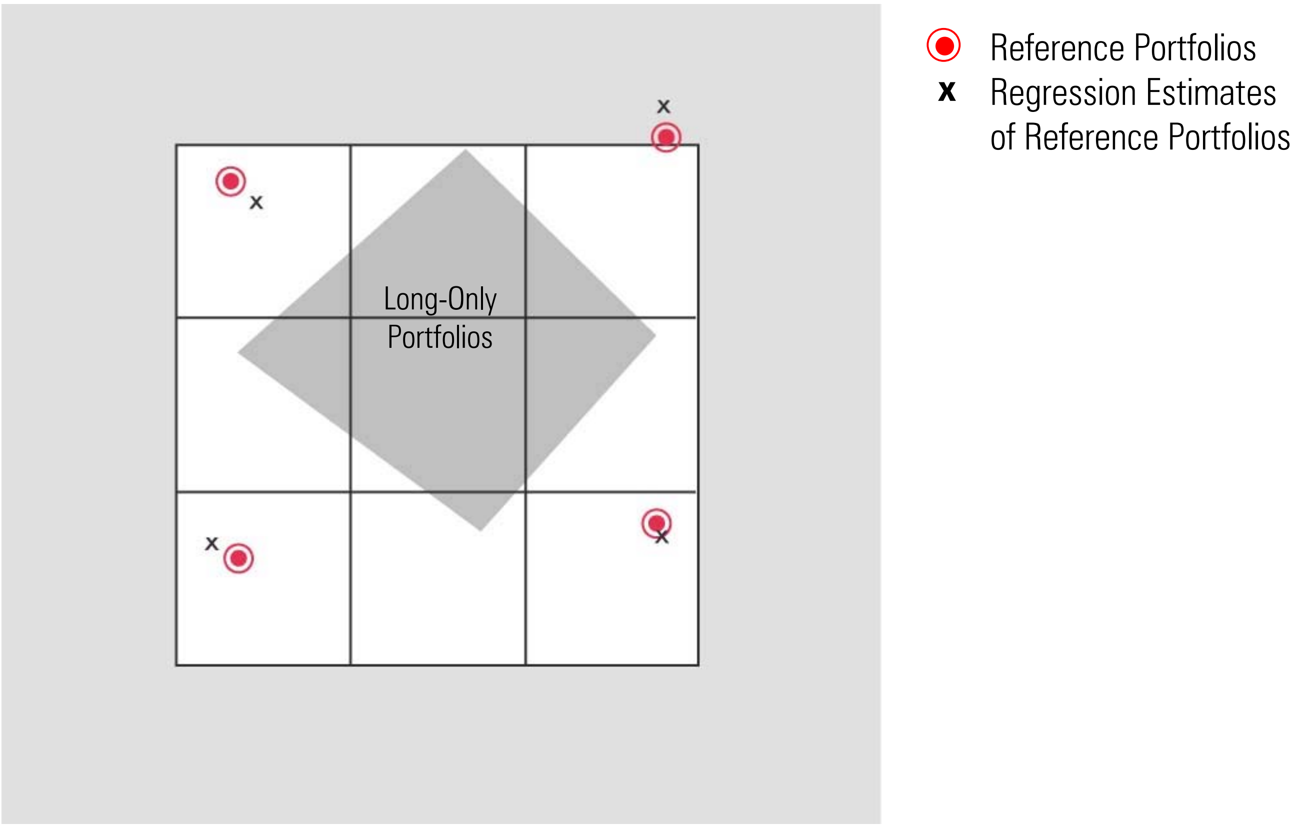


Exhibit 9: Number of Distinct Funds in Each Category

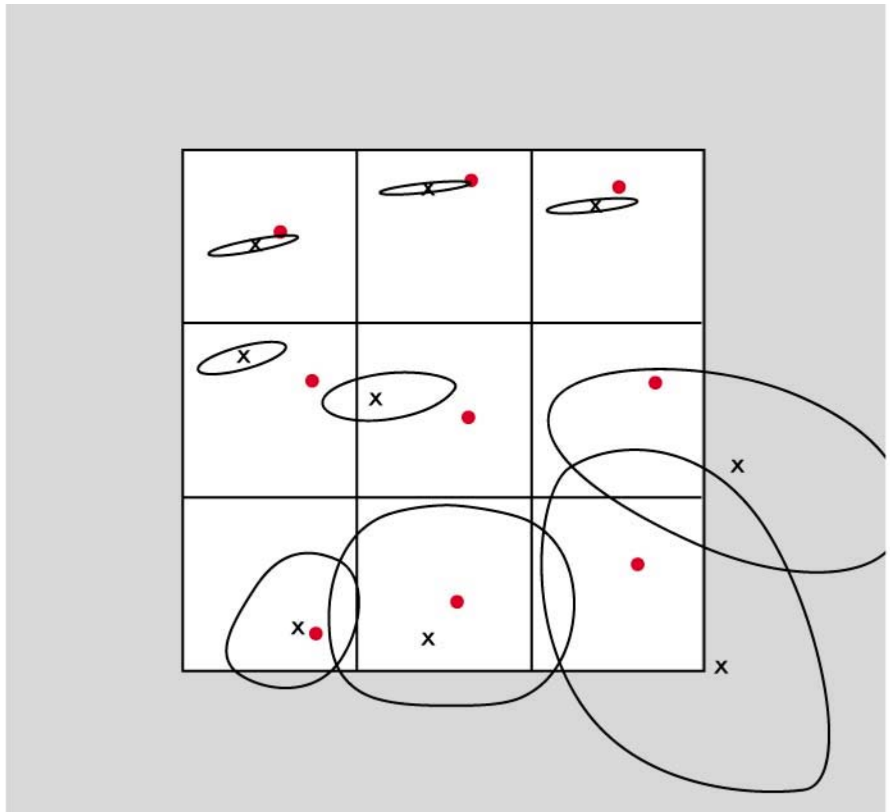
	Value	Blend	Growth
Large-Cap	273	438	354
Mid-Cap	91	86	222
Small-Cap	97	124	224

Exhibit 10: R-Squareds for Category Averages

Monthly Data 2000-2002

	Value	Blend	Growth
Large-Cap	98.7%	99.6%	99.0%
Mid-Cap	98.2%	97.6%	94.1%
Small-Cap	97.5%	95.3%	94.8%

Exhibit 11: Estimated Centroids for Category Averages



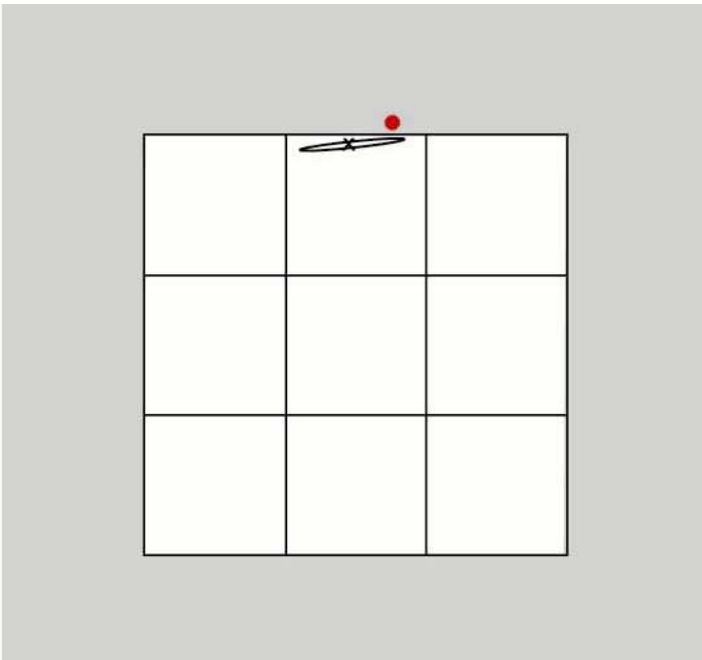
Holdings-Based

Returns-Based and 95% Confidence Region

Centroids of Reference Portfolios (Average over 2000-2002)

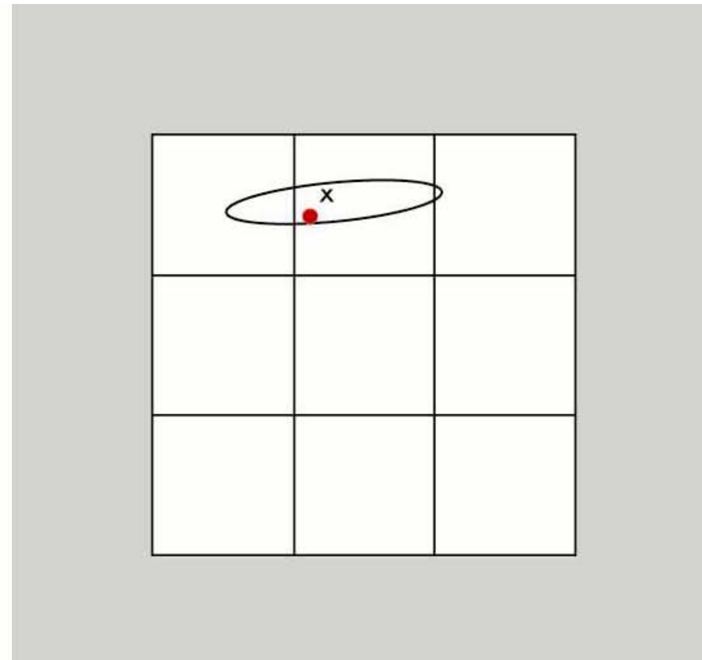
Exhibit 12: Examples of Similar Fund Results

Fidelity Magellan



$R^2 = 99.1\%$

Legg Mason Value



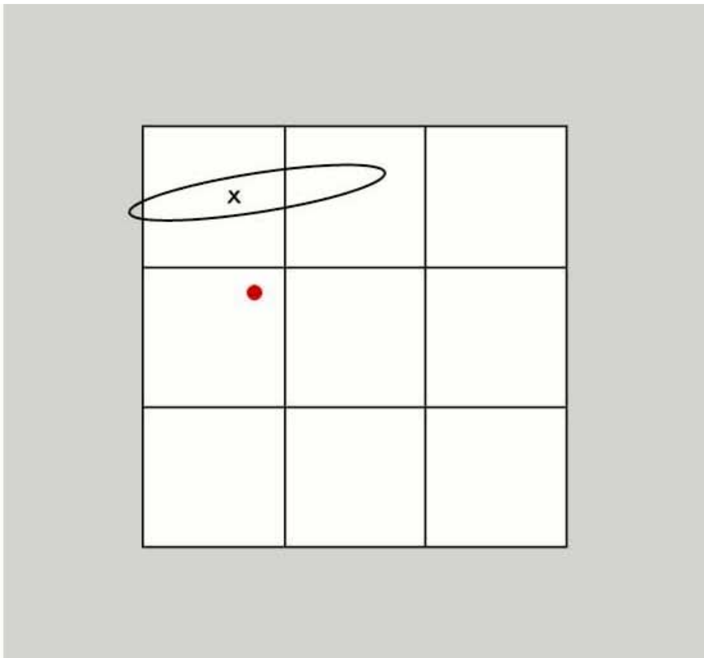
$R^2 = 88.3\%$

● Holdings-Based

ⓧ Returns-Based and
95% Confidence Region

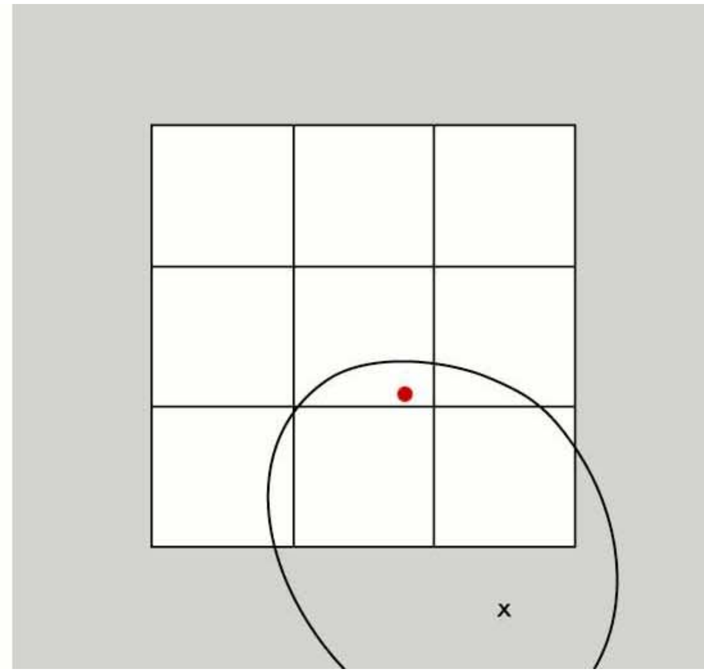
Exhibit 13: Examples of Substantially Different Fund Results

Oakmark Select I



$R^2 = 79.5\%$

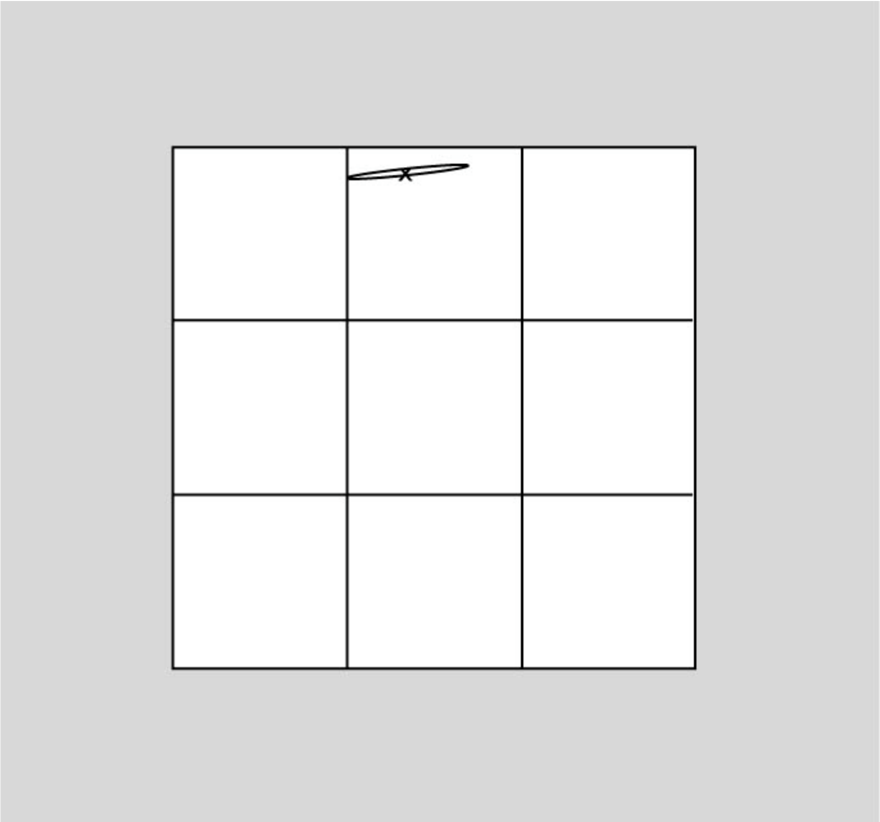
Van Kampen American Value



$R^2 = 89.1\%$

- ● Holdings-Based
- ⓧ Returns-Based and 95% Confidence Region

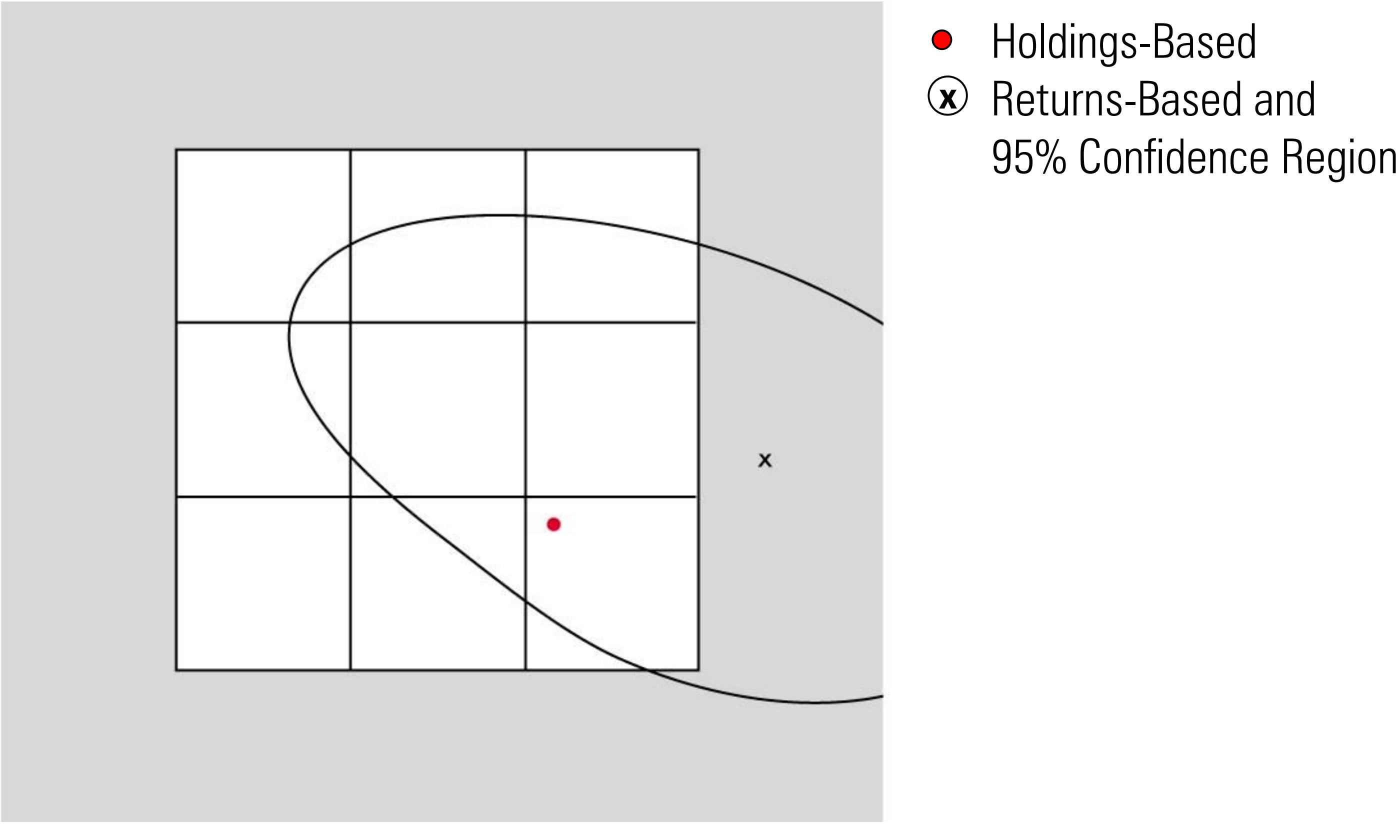
Exhibit 14: Returns-Based Analysis for a Fund that Uses Futures



ⓧ Estimated Centroid and 95% Confidence Region

PIMCO StocksPlus ($R^2 = 98.4\%$)

Exhibit 15: Example of Results for a Fund that Shorted Stocks



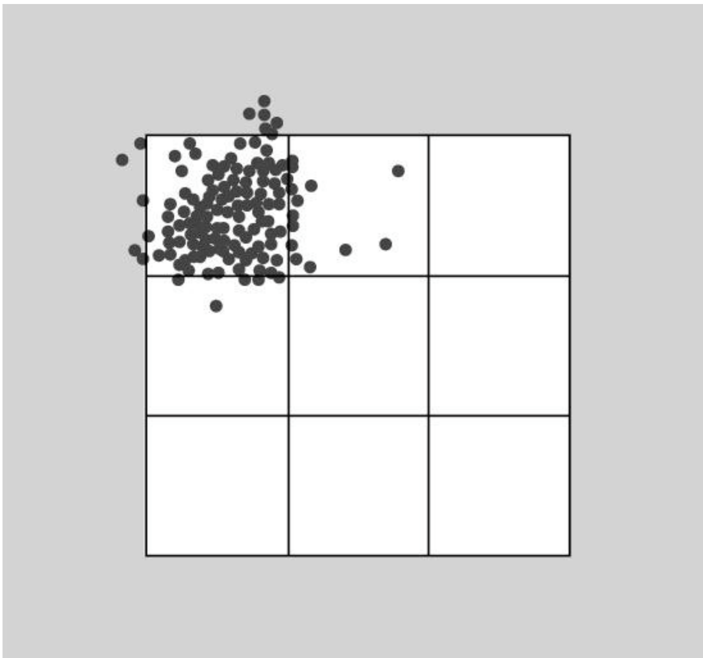
Needham Growth ($R^2 = 79.4\%$)

Exhibit 16: Category Average R-Squareds

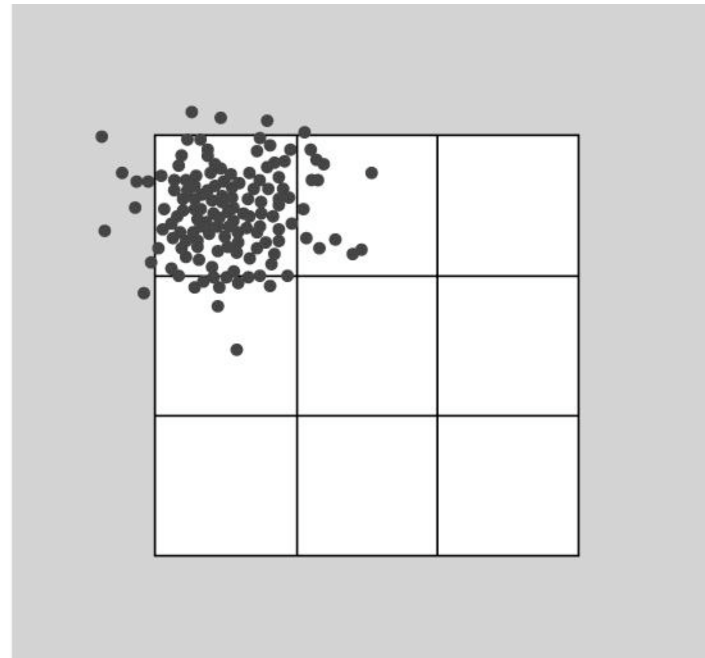
Monthly Data 2000-2002

	Value	Blend	Growth
Large-Cap	91.2%	93.1%	90.8%
Mid-Cap	85.1%	85.0%	85.6 %
Small-Cap	84.2%	83.6%	85.9%

Exhibit 17: Centroid Estimates for Large-Cap Value Funds



Holdings-Based

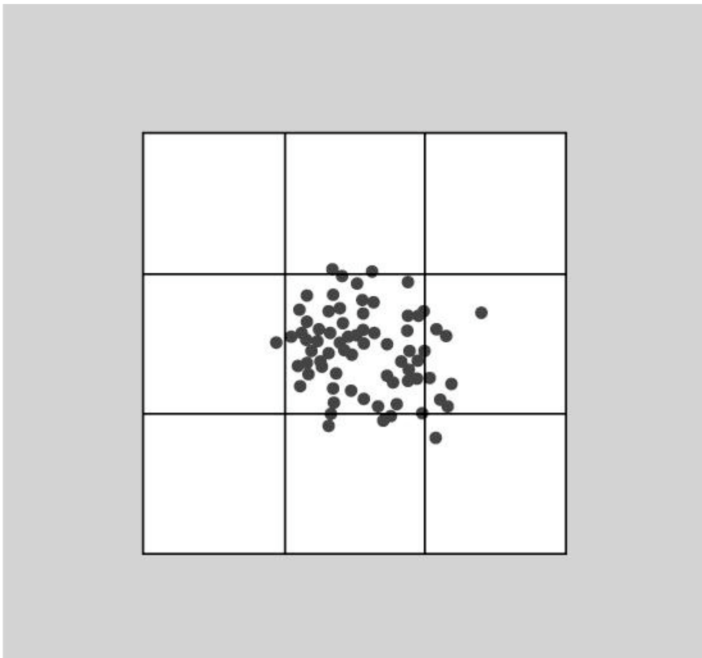


Returns-Based

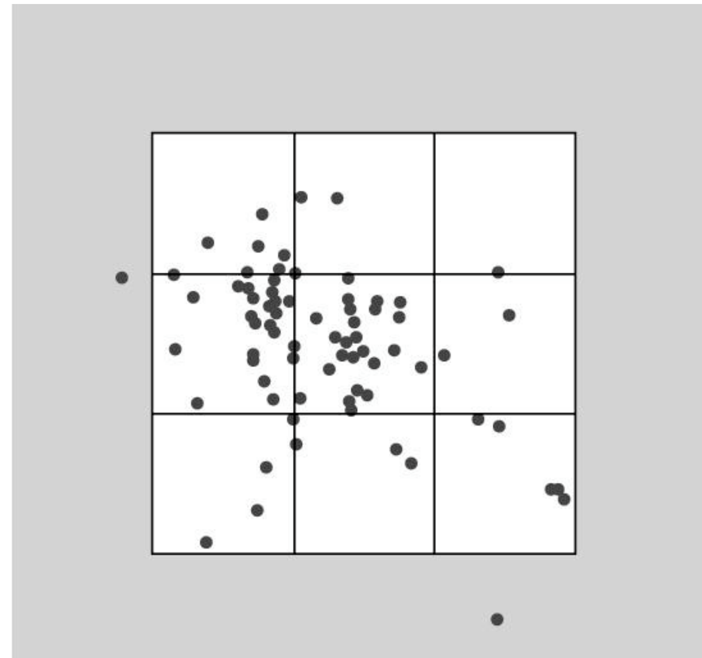
Coordinate*	x	y
Correlation	59.4%	57.4%

* Rescaled as described in Morningstar [2003].

Exhibit 18: Centroid Estimates for Mid-Cap Blend Funds



Holdings-Based

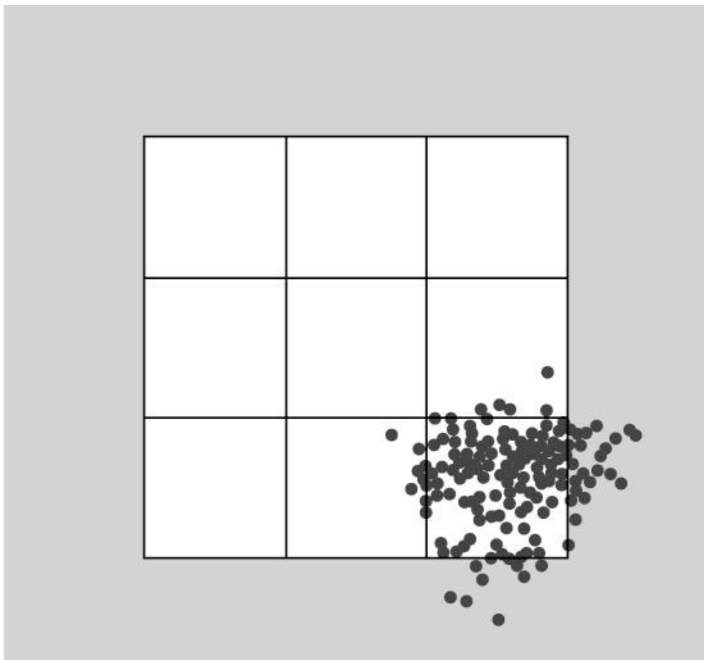


Returns-Based

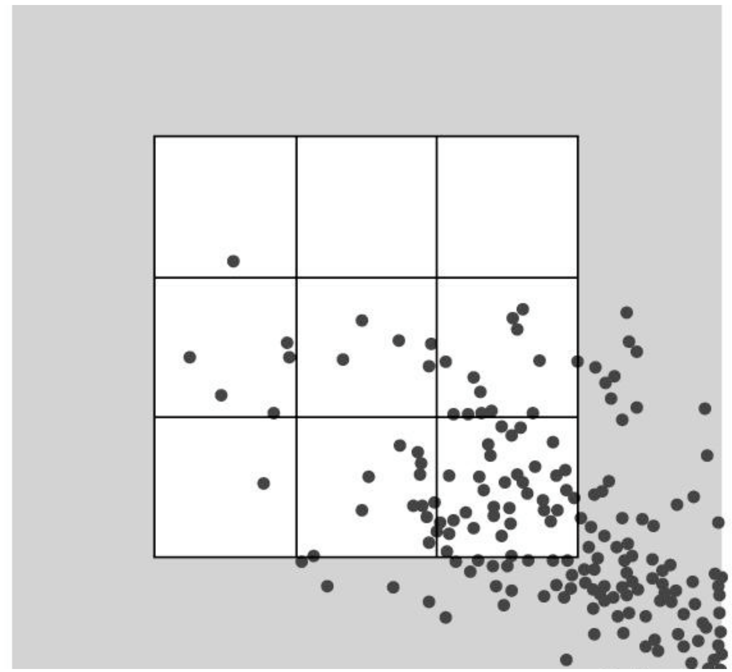
Coordinate*	x	y
Correlation	63.6%	61.9%

* Rescaled as described in Morningstar [2003].

Exhibit 19: Centroid Estimates for Small-Cap Growth Funds



Holdings-Based

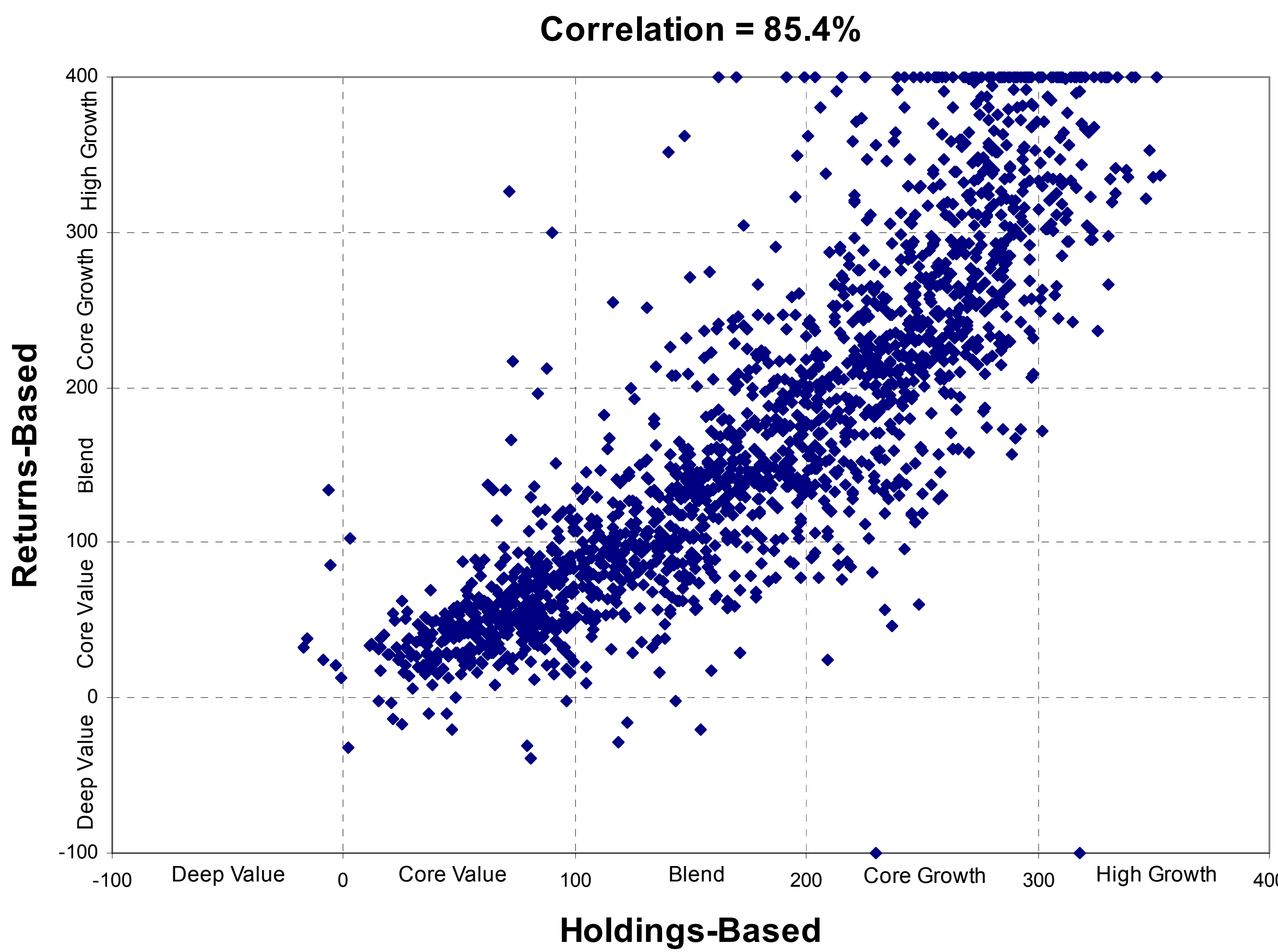


Returns-Based

Coordinate*	x	y
Correlation	56.8%	15.5%

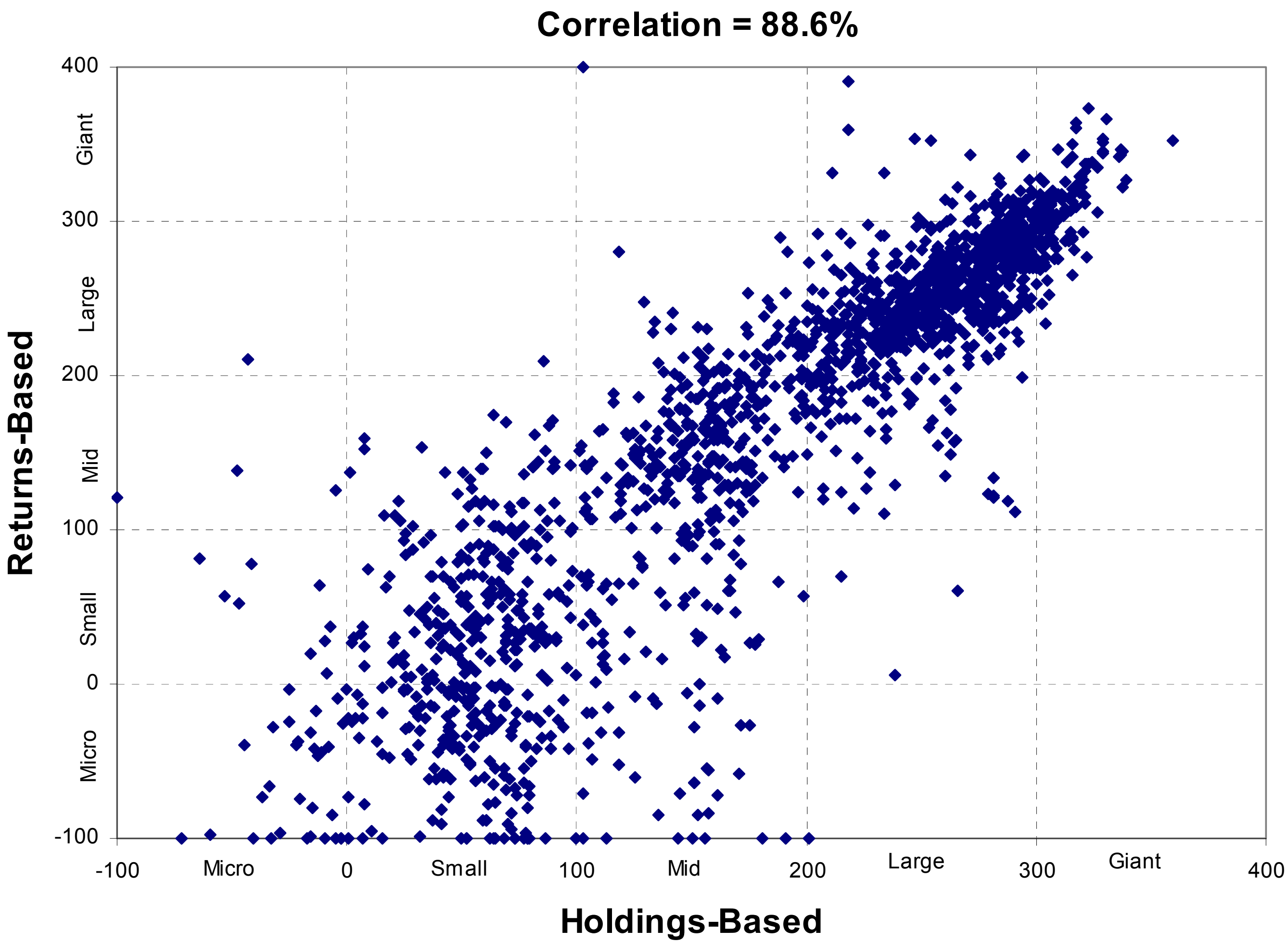
* Rescaled as described in Morningstar [2003].

Exhibit 20: Holdings-Based vs. Returns-Based X-Coordinates*



* Rescaled as described in Morningstar [2003].

Exhibit 21: Holdings-Based vs. Returns-Based Y-Coordinates*



* Rescaled as described in Morningstar [2003].

Exhibit 22: Example Time-Varying Reference Portfolio Allocations

When Best Performing Ref. Port. is:	% Large Value	% Large Growth	% Small Value	% Small Growth	% of Time
Large Value	75.00	8.33	8.33	8.33	25.00
Large Growth	0.00	100.00	0.00	0.00	16.67
Small Value	12.50	12.50	62.50	12.50	33.33
Small Growth	8.33	8.33	8.33	75.00	25.00
Average	25.00	25.00	25.00	25.00	

Exhibit 23: Regression Results for Style Inconsistency Example

Parameter	Estimate	T-Stat.
α	2.70%	11.02
β_{LV}	27.43%	2.71
β_{LG}	41.22%	4.53
β_{SV}	8.13%	0.80
β_{SG}	21.99%	3.11

$R^2 = 95.35\%$

Standard Error of Regression = 1.41%